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3.2. COROLLARY. Let T be a contradiction on a Banach space B, and assume that B^* is a G-space. Then (c) implies (a).

Proof. If T^* is strongly ergodic, then 3.1 implies that T^{**} is strongly ergodic. Since B is a norm-closed subspace of B^{**}, it follows that T is strongly ergodic.

3.3. Example. To show that the hypothesis in 3.2 is really needed, we give an example where T^* is strongly ergodic while T is not. Define T on c_o (= null sequences) by $(Tx)_1 = x_1$ and $(Tx)_n = x_{n-1}$ for $n > 1$. For each $x \in c_o$, the sequence $\{ (T^k x)_. : k \geq 1 \}$ converges to $(x_1, x_1, x_1, \dots)$, which is not in c_o if $x_1 \neq 0$. It is easy to check that the adjoint $S = T^*$ is given for $y \in \ell^1$ by

$$Sy = (y_1 + y_2, y_3, y_4, \dots).$$

The iterates $S^k y$ converge pointwise and in ℓ^1-norm to $(\Sigma_i y_i, 0, 0, \dots)$. (Note that the projection so defined is norm but not weak-* continuous on ℓ^1.)

It is apparently not known whether there exist non-reflexive spaces such that both B and B^* are G-spaces [D, page 105] , so it is not clear whether 3.1 and 3.2 have non-trivial joint applications.

4. *Final Remark.* Corollary 1.4 has been proved independently by Michael Lin, for a semigroup of contractions. His proof embeds the π-invariant vectors in the dual of the π^*-invariant vectors. I am grateful to Lin, as well as to Robert Sine, for correspondence on this and other matters.

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