and only if Y is small as an R module.

b) If P has a projective cover as a Δ module, then P is Δ projective.

Proof of a). Since $\Delta/J(\Delta) = \Delta/J(R)\Delta$ is Artinian, $J({}_{\Delta}P) = J(\Delta)P = J(R)P = J({}_{R}P)$. Now $J({}_{R}P)$ is R small in P; hence $J({}_{\Delta}P)$ is Δ small in P. See [1, Proposition 4]. We assume Y is Δ small, hence Y is contained in every maximal left Δ module. Thus $Y \subseteq J({}_{\Delta}P)$, which is R small.

Proof of b). Let $f: Q \to P \to 0$ be the Δ cover of P. Then the kernel of f is Δ small, hence by 1) ker f is R small. Since P is R projective, f splits. Thus ker $f = 0$. Hence P is Δ projective.

COROLLARY. *Every left Δ module P which has a Δ cover and when viewed as an R module is semiperfect, is Δ projective.*

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