

## Werk

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to be an easy task. Even in the commutative case, we need much more than mere commutativity. If  $\mathcal{Q}$  is a commutative  $C^*$ -algebra with identity, then its spectrum is automatically compact  $T_2$ , but in general it can be very "unsmooth". (See [8] p. 273 for an example of extreme pathological behavior). The proof of Kuchnirenko's theorem uses very strongly the fact that the spectrum of a bounded classical system is a compact  $C^\infty$ -manifold. In any case, we wish to give a rather trivial example of this theorem in a quantum mechanics context which covers finite lattices or a system of a finite number of spins :

**5.2. THEOREM.** *If  $\mathcal{H}_\rho$  is finite-dimensional,  $b(T)$  is finite.*

*Proof.* Let  $\dim \mathcal{H}_\rho = N$ . If  $N=1$ ,  $b(\alpha) = 0$  for any  $\alpha$ , so the case  $N=1$  is trivial. Assume  $N > 1$ . From Proposition 2.1 we have that  $b(\alpha) \leq \log_2 n$  where  $n$  is the cardinality of  $\alpha$ . But  $n \leq N$ , so  $b(\alpha) \leq \log_2 N \leq N$ . By Corollary 2.3 and Lemma 3.1

$$b(\alpha \vee T\alpha \vee \dots \vee T^{k-1}\alpha) \leq \sum_{i=0}^{k-1} b(T^i\alpha) = \sum_{i=0}^{k-1} b(\alpha) \leq Nk$$

Thus  $b(\alpha, T) \leq N$ . Q.E.D.

We wish to propose the following question as an unsolved problem to this date: What are the weakest conditions on  $\mathcal{Q}$  that guarantee that  $b(T)$  is finite?

#### References

1. S. M. Moore : *An Algebraic Formulation of Ergodic Problems*. Rev. Col. Mat. 9, 189-204 (1975).
2. A. N. Kolmogorov : *A New Metric Invariant of Transitive Dynamic Systems and Automorphisms of Lebesgue Fields*. Doklady Akad. Nauk 119 No. 5, 861-864 (1958).
3. V. I. Arnold, A. Avez : *Ergodic Problems of Classical Mechanics*. W.A. Benjamin, New York, 1968.
4. W. Rudin : *Real and Complex Analysis*. McGraw-Hill, New York, 1966.
5. V.A. Roblin, Ya. Sinai : *Construction and Properties of Invariant Measurable*

- Partitions*. Doklady Akad. Nauk. 141 No. 6, 1038-1041 (1961). (Also in English : Sov. Math. Doklady 2 No. 6, 1611-1614 (1961)).
6. A. N. Kolmogorov : *On the Entropy per Unit Time as a Metric Invariant of Automorphisms*. Doklady Akad. Nauk. 124 No. 4, 754-755 (1959).
  7. A. G. Kuchnirenko : *An Estimate from Above for the Entropy of a Classical System*. Doklady Akad. Nauk. 161 No. 1, 37-38 (1965). (Also in English : Sov. Math. Doklady 6 No. 2, 360-362 (1965).)
  8. W. Rudin : *Functional Analysis*, McGraw-Hill, New York, 1973.

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