then by applying Π_H to this surface we would have a surface in P which spanned $C_1 \cup C_2$ but which dit not contain the point w_1. This contradicts the fact that w_1 is inside the simple closed curve $C_1 \cup C_2$.

Let K be any cone made up of all the line segments connecting a fixed point p on $f(C_2)$ to all other points on $f(C_2)$. It was pointed out above that K is the set of points lying on a surface which spans $f(C_2)$. Applying Lemma 1 with $A = S_i$, we see that $f(C_2)$ links with $f(S_i)$. Consequently, there exists $x_i \varepsilon S_i$ for which $f(x_i) \varepsilon K$. Since $B(x_1, \rho) \subset B(r_0)$, $f(B(x_i, \rho)) \quad f(C_2) = \phi$. This follows since $f(B(r_0)) \quad f(\partial B(r_0)) = \phi$. Now we apply (L') and conclude that $B(f(x_i), m\rho) \subset f(B(x_i, \rho))$. Thus $f(C_2) \quad B(f(x_i), m\rho) = \phi$. Let L_i be the straight line containing p and $f(x_i)$. Then by the definition of x_i and K, there is a point $q_i \varepsilon L_i \quad f(C_2)$ which lies on the opposite side of $B(f(x_i), m\rho)$ from p. We now apply Lemma 2 with $B_i = B(f(x_i), m\rho)$ and conclude that $\lambda(f(C_2)) \geq (4 + 2\pi) m\rho$. However, since $\lambda(C_2) < (4 + \pi)\rho$, we have, by (U'), that $\lambda(f(C_2)) < M(4 + \pi)\rho$. Putting these two bounds for $\lambda(f(C_2))$ together we obtain $M/m > \frac{4 + 2\pi}{4 + \pi}$, which is exactly what we had to prove.

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