ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

CONVERGENCE CRITERION FOR MULTIPARAMETER STOCHASTIC PROCESSES

Petr Lachout (MFF UK, Sokolovská 83, 18600 Praha 8, Czechoslovakia), received 1.7. 1987)

Bickel and Wichura [1] extended the tightness criterion from processes on $D\langle 0,1)$ (see Billingsley [2]) to processes on $D\langle 0,1)^k$, $k>1$. However, they impose an additional condition that the processes should vanish along the lower boundary of $\langle 0,1\rangle^k$. This means that their criterion does not apply to many empirical processes of interest.

We shall provide an improved tightness criterion for processes in $D\langle 0,1)^k$ without the above additional condition.

Definition: Let $k\in N$, $d=1,\dots,k$, $j=0,\dots,k-d$, $\varphi:\langle 0,1\rangle^k\rightarrow\langle 0,1\rangle^k$ be a permutation of coordinates and $X=(X(t),\ t\in\langle 0,1\rangle^k$ be a random process. Define

(1) $$\Delta X(d,j,\varphi)\Big(\mathop{\mathsf{X}}_{i=1}^{d}\langle a_i,b_i)\Big)= \sum_{i=1}^{d}\ \sum_{\delta_i=a_i,b_i}(-1)^{\Sigma_p I(\delta_p=a_p)}\, X\circ\varphi(\delta_1,\dots,\delta_d,0,\dots,0,\underbrace{1,\dots,1}_{j\times})$$

for every $0\le a_i<b_i\le 1$, $i=1,\dots,d$.

We shall prove the following theorem.

Theorem: Let $X=(X(t),\ t\in\langle 0,1\rangle^k)$, $k\in N$, be a random process right-continuous in every coordinate. Let $\mu_{d,j,\varphi}$, $d=1,\dots,k$, $j=0,\dots,k-d$ and $\varphi:\langle 0,1\rangle^k\rightarrow\langle 0,1\rangle^k$ being a permutation of coordinates, be a bounded measure with continuous marginals.

If there exists $\alpha,\beta>0$ such that

(2) $$P(|\Delta X(d,j,\varphi)(A)|>y,\ |\Delta X(d,j,\varphi)(B)|>y)\le y^{-\alpha}\mu_{d,j,\varphi}(A\cup B)^{1+\beta}$$

holds for every $y>0$, $d=1,\dots,k$, $j=0,\dots,k-d$ and every permutation φ and for all

$$A=\mathop{\mathsf{X}}_{i=1}^{d}\langle a_i,b_i),\quad B=\mathop{\mathsf{X}}_{i=1}^{d}\langle g_i,h_i),$$

$A\cap B=\emptyset$, clo $A\cap$ clo $B\neq\emptyset$, then there exist an absolute constant $Q>0$ and a function $R:(0,1\rangle\rightarrow\langle 0,1\rangle$, $\lim_{\varepsilon\to 0_+}R(\varepsilon)=0$, such that

(3) $$P(\sup\{\min\{|X\circ\varphi(t,u)-X\circ\varphi(s,u)|,\ |X\circ\varphi(s,u)-X\circ\varphi(v,u)|\}\ |\ 0\le t<s<v\le 1,\ v-t<\varepsilon,\ u\in\langle 0,1\rangle^{k-1},\ \varphi \text{ is a permutation of coordinates}\}>y)\le Qy^{-\alpha}R(\varepsilon)$$ for every $\varepsilon\in(0,1\rangle$.

If $k=1$ then the criterion (2) reduces to the criterion in Billingsley [2] (see Theorem 15.6) while it is an improvement of the criterion of [1] if $k>1$.

References:

[1] Bickel P.J., Wichura M.S.: Convergence criteria for multiparameter stochastic processes and some applications, The Annals of Mathematical Statistics 42(1971), 1656-1670.

[2] Billingsley P.: Convergence of Probability Measures, John Wiley, New York, 1968.

IDENTITIES FOR DIRECT DECOMPOSABILITY OF CONGRUENCES CAN BE WRITTEN IN TWO VARIABLES

Jaromír Duda (Kroftova 21, 616 00 Brno, Czechoslovakia), received 2.10. 1987

A variety V has directly decomposable congruences if every congruence relation on the product $A\times B$ of algebras $A,B\in V$ is uniquely determined by its projections onto A and B.

Varieties with directly decomposable congruences form a Mal´cev class. The known identities contain at least three variables. We state that two variables are enough.

Theorem. For a variety V the following conditions are equivalent:

(1) V has directly decomposable congruences;

(2) There exist binary terms $r_1,\dots,r_m,s_1,\dots,s_m,t_1,\dots,t_m$ and (2+m)-ary terms $d_1,\dots,d_n$ such that V satisfies

$x=d_i(y,y,r_1(x,y),\dots,r_m(x,y))$, $1\le i\le n$,

$x=d_1(x,y,s_1(x,y),\dots,s_m(x,y))$,

$y=d_1(x,y,t_1(x,y),\dots,t_m(x,y))$,

$d_i(y,x,s_1(x,y),\dots,s_m(x,y))=d_{i+1}(x,y,s_1(x,y),\dots,s_m(x,y))$, $1\le i< n$,

$d_i(y,x,t_1(x,y),\dots,t_m(x,y))=d_{i+1}(x,y,t_1(x,y),\dots,t_m(x,y))$, $1\le i< n$,

$y=d_n(y,x,s_1(x,y),\dots,s_m(x,y))$,

$y=d_n(y,x,t_1(x,y),\dots,t_m(x,y))$.

LANDESMAN-LAZER CONDITION FOR PERIODIC PROBLEMS WITH JUMPING NONLINEARITIES

P. Drábek (Katedra matematiky VŠSE, Nejedlého sady 14, 30614 Plzeň, Czechoslovakia), received 13.10. 1987

Let us consider the periodic boundary value problem at resonance

(1) $x''(t)+m^2x(t)+g(t,x(t))=e(t), x(0)-x(2\pi)=x'(0)-x'(2\pi)=0,$

$m\ge 0$ is an integer, $e\in L^1(0,2\pi)$. We assume that g is a Carathéodory´s function satisfying the growth restriction

$$|g(t,x)|\le p(t)+c|x|;$$

for a.e. $t\in[0,2\pi]$, all $x\in R$ with $c>0$ and $p\in L^1(0,2\pi)$. Moreover, assume that $g_+(t)=\liminf_{x\to+\infty} g(t,x)$ and $g_-(t)=\limsup_{x\to-\infty} g(t,x)$. We impose the following restriction on the growth of g. Let for a.e. $t\in[0,2\pi]$,

$0\le\limsup_{x\to+\infty} x^{-1}g(t,x)\le a-m^2$ and $0\le\limsup_{x\to-\infty} x^{-1}g(t,x)\le b-m^2$

with strict inequality on the set of positive measure in $[0,2\pi]$, where

$$a^{-1/2}+b^{-1/2}=2(m+1)^{-1}.$$

Theorem. Assume that g satisfies all the assumptions stated above. Then the periodic problem (1) has at least one solution provided that

$$\int_0^{2\pi} e(t)v(t)dt<\int_{v>0} g_+(t)v(t)dt+\int_{v<0} g_-(t)v(t)dt$$

for all $v\in \mathrm{Span}\{\sin mt,\cos mt\}\setminus\{0\}$.

Remark 1. Note that our assumptions laid on g are satisfied also in the case when g is "jumping" over eigenvalues different from m^2. In this direction our results generalize the previous ones (see [1]).

Remark 2. By the same approach used for the periodic problem we can prove analogous existence results for the two-point boundary value problem.

Reference

[1] R. Iannacci and M.N. Nkashama, Unbounded perturbations of forced second order ordinary differential equations at resonance, J. Differential Equations 69(1987), 289-309.