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REMARKS ON INFLATED MAPPINGS

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Abstract: Organizing centre of an imperfect bifurcation problem $F(u,\lambda,\alpha)=0$ is related to a simple root of an auxiliary operator (= the inflated mapping). The construction of an inflated mapping depends on a classification of the organizing centre.

Key words: Imperfect bifurcation problems, organizing centre, numerical approximation.

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1. Introduction. Let U and Y be Banach spaces. We consider an operator $F:U\times\mathbb{R}_1\times\mathbb{R}_k\longrightarrow Y$. The variable x of $F=F(x)$ is a triple $x=(u,\lambda,\alpha)$, where (in a bifurcation context) u and λ and α respectively are the state variable and the control parameter and the parameter of an imperfection.

A point $x_0=(u_0,\lambda_0,\alpha_0)\in U\times\mathbb{R}_1\times\mathbb{R}_k$ is called the singular point of F if

(1.1) $$F(x_0)=0$$

(1.2) $$\dim \operatorname{Ker} F_u(x_0)=m\geq 1,$$

where F_u denotes the partial Fréchet derivative of F (at x_0) w.r.t. the variable u, and Ker $F_u(x_0)$ is the kernel of $F_u(x_0):U\longrightarrow Y$.

Moreover, we assume

$F_u(x_0):U\longrightarrow Y$ to be Fredholm with index zero

and

$F\in C^{\infty}(X,Y)$, where X is a neighbourhood of x_0.

Let us consider an operator

$L:U\longrightarrow\mathbb{R}_m$ linear, bounded

satisfying the following implication:

(1.3) $$\begin{cases}\text{if } v\in \operatorname{Ker} F_u(x_0) \text{ and } Lv=0\\ \text{then } v=0.\end{cases}$$

Choose a basis $\{a_1,\dots,a_m\}$ of $\mathbb{R}_m$. Then the condition (1.2) can be reformulated as follows:

(1.4) for each $i=1,\dots,m$ there exists $v_i^{(1)} \in U$:

$$F_u(x_0)v_i^{(1)}=0,\ Lv_i^{(1)}=a_i .$$

Note that if $\dim U \geq m$ then the property (1.3) is a generic property on the class of all linear bounded operators $L:U \to \mathbb{R}_m$.

The conditions (1.1), (1.4) do not define x_0 uniquely. In general, a point x_0 satisfying (1.1), (1.4) is not isolated. To make it isolated, we have to require more then (1.1), (1.4): If x_0 is an organizing centre of F (i.e. "the most singular" point which is locally available) then there is a chance for x_0 to be locally unique.

In this paper, we are trying to suggest a way how to formulate necessary and sufficient conditions on x_0 to be an "organizing centre". The important point is that these conditions are stated in terms of F (and its partials). We hint at numerical applications of this procedure in Section 5.

We quote the papers [3],[4],[5], dealing with the same idea. Our approach is stimulated by the preprint [1].

2. Classification of singular points. Following [2], we review basic ideas of Liapunov-Schmidt reduction and classification of germs of smooth mappings in the context of an imperfect bifurcation.

Define a projection

$$\Pi : U \to \operatorname{Ker} F_u(x_0)$$

fulfilling the following implication: if $u \in U$ then $\Pi u = v \in \operatorname{Ker} F_u(x_0)$ and $Lv = Lu$. Let Π^c be the complement of Π, i.e.,

$$\Pi^c = I - \Pi \quad (I \text{ is the identity } U \to U).$$

We set $W = \Pi^c(U)$, i.e.,

$$W = \{v \in U : Lv = 0\}.$$

Obviously, W is closed and

$$U = \operatorname{Ker} F_u(x_0) \oplus W.$$

Remind that $F_u(x_0)$ is assumed to be Fredholm with index zero. Let $\mathcal{R}(F_u(x_0))$ denote the range of $F_u(x_0)$. There exists a projection

$$Q : Y \to \mathcal{R}(F_u(x_0)).$$

Let Q^c be its complement, i.e.,

$$Q^c = I - Q \quad (I \text{ is the identity } Y \to Y).$$

Then

$$Y=\mathcal{R}(F_u(x_0))\oplus Q^c(Y)$$

where both components are closed and

$$\dim Q^c(Y)=\dim \operatorname{Ker} F_u(x_0).$$

Thus, for each $r\in Y$, there exists the unique $z\in U$ such that

$$F_u(x_0)z=Qr,\ Lz=0.$$

We set $F_u^+(x_0)r=z$. Then

(2.1)
$$F_u^+(x_0):Y\longrightarrow W$$

is linear, bounded.

The condition (1.1) can be reduced to a so called bifurcation equation, see the coming (2.3): If $(v,\lambda,\alpha)\in \operatorname{Ker} F_u(x_0)\times \mathbb{R}_1\times \mathbb{R}_k$ then we define $w\in U$:

(2.2)
$$\begin{cases} QF(w+v,\lambda,\alpha)=0 \\ Lw=0 \text{ (i.e., } w\in W). \end{cases}$$

By means of the Implicit Function Theorem,

$$w=w(v,\lambda,\alpha),\ w\in C^\infty(\mathcal{V},W)$$

where $\mathcal{V}\subset \operatorname{Ker} F_u(x_0)\times \mathbb{R}_1\times \mathbb{R}_k$ is a sufficiently small neighbourhood of the point (v_0,λ_0,α_0), $v_0=\Pi u_0$. To be precise, there exists a neighbourhood $\mathcal{W}$ of $\Pi^c u_0$ (in W) such that (2.2) is satisfied for $w\in\mathcal{W}$ and $(v,\lambda,\alpha)\in\mathcal{V}$ if and only if $w=w(v,\lambda,\alpha)$. Thus, we define

$$\mathcal{U}=\{(u,\lambda,\alpha):(\Pi u,\lambda,\alpha)\in\mathcal{V},\ \Pi^c u\in\mathcal{W}\}.$$

It can be easily concluded that

$$F(u,\lambda,\alpha)=0,\ (u,\lambda,\alpha)\in\mathcal{U}$$

if and only if

(2.3)
$$g(v,\lambda,\alpha)=0,\ (v,\lambda,\alpha)\in\mathcal{V}$$

where

(2.4)
$$g(v,\lambda,\alpha)=Q^cF(v+w(v,\lambda,\alpha),\lambda,\alpha).$$

Both $\operatorname{Ker} F_u(x_0)$ and $Q^c(Y)$ can be identified with $\mathbb{R}_m$. Then g could be understood as a germ of C^∞-mapping

$$g:\mathbb{R}_m\times\mathbb{R}_1\times\mathbb{R}_k\longrightarrow\mathbb{R}_m$$

centred at (v_0,λ_0,α_0).

Let us proceed with ideas of classification. Assume the space of all germs h of C^∞-mappings

$$h:\mathbb{R}_m\times\mathbb{R}_1\longrightarrow\mathbb{R}_m$$

centred at (v_0, λ_0). An equivalence (so called contact equivalence) is defined on this space; the equivalence preserves important topological properties of bifurcation diagrams. The equivalence classes are called orbits. If a germ $h=h(v,\lambda)$ has a finite codimension then the relevant orbit is a semialgebraic variety of a finite codimension in the linear space of Taylor coefficients (i.e. the space of all partials of h at (v_0, λ_0).

Just two examples:

Example 1. Assume m=1, and define

$$G=(h,h_v,h_\lambda,h_{vv},h_{v\lambda})^T: \mathbb{R}_1 \times \mathbb{R}_1 \longrightarrow \mathbb{R}_5.$$

If G=0 at (v_0, λ_0) and some "nondegeneracy conditions" hold (namely, $h_{vvv} \neq 0$, $h_{\lambda\lambda} \neq 0$) then (v_0, λ_0) is called the winged cusp singularity, see [2], p. 198.

Example 2. Assume m=2, and define $G=(h,h_v)^T: \mathbb{R}_2 \times \mathbb{R}_1 \longrightarrow \mathbb{R}_6$. If G=0 at (v_0, λ_0) and some nondegeneracy conditions hold (e.g. $h_\lambda \neq 0$) then (v_0, λ_0) is called the hilltop bifurcation point, see [2], p. 403.

Each particular singularity (v_0, λ_0) has to satisfy a set of ℓ algebraic conditions

$$G=0 \text{ at } (v_0, \lambda_0)$$

where $G: \mathbb{R}_m \times \mathbb{R}_1 \longrightarrow \mathbb{R}_\ell$; ℓ is finite if h has a finite codimension.

The germ $g=g(v,\lambda,\alpha)$ can be viewed as a perturbation of h. Naturally, we replace h by g in the particular definition of G. Then

(2.5) $$G: \mathbb{R}_m \times \mathbb{R}_1 \times \mathbb{R}_k \longrightarrow \mathbb{R}_\ell$$

and the condition on a singular point reads as

(2.6) $$G=0 \text{ at } (v_0, \lambda_0, \alpha_0).$$

The condition (2.6) defines $(v_0, \lambda_0, \alpha_0)$ locally uniquely if and only if

(A) $$\begin{cases} m+1+k=\ell \\ \text{Jacobian of } G \text{ at } (v_0, \lambda_0, \alpha_0) \text{ does not vanish.} \end{cases}$$

At this place, we can formulate the following conjecture: The condition (A) is equivalent to the assumption that $g=g(v,\lambda,\alpha)$ is a universal unfolding of the germ $g(\cdot,\cdot,\alpha_0): \mathbb{R}_m \times \mathbb{R}_1 \longrightarrow \mathbb{R}_m$. In such a case, $k=\operatorname{codim} g(\cdot,\cdot,\alpha_0)$. Note that if the codimension $k \leq 3$ then there is a finite choice of mappings G. Let us quote 2 , Theorem 2.1, p. 400, where the relevant G's are listed.

The aim of this paper is to indicate how to formulate (2.6) in terms of

F (and its partial derivatives w.r.t. u and λ) at the singular point x_0.

3. Construction of inflated mappings. In order to illustrate the idea, we assume the following examples:

Case 1: $G=(g,g_v,g_\lambda)^T$;

Case 2: $G=(g,g_v,g_{vv})^T$;

Case 3: $G=(g,g_v,g_\lambda,g_{vv},g_{v\lambda})^T$;

there is no restriction on dimension m. Conditions G=0 classify singularities (v_0,λ_0,α_0) in the sense of the previous section.

For each of the above cases, we derive the equivalent conditions on (u_0,λ_0,α_0). It will appear that (u_0,λ_0,α_0) is related to a root of an operator $\mathcal{F}$, where $\mathcal{F}$ is constructed by means of F and its partials w.r.t. u and λ. Let us say that $\mathcal{F}$ is the inflated mapping corresponding to F.

Notation: If it is not stated otherwise then the values of F and its derivatives are understood at the singular point $x_0=(u_0,\lambda_0,\alpha_0)$. Similarly, the operators w and g (and their derivatives) are evaluated at the "projected" x_0, i.e. at (v_0,λ_0,α_0).

First, let us remind our assumption on x_0, see (1.1) and (1.4). It reads as follows:

(3.1) $$F=0$$

(3.2) $$\exists v_i^{(1)}\in U,\ i=1,\dots,m: F_u v_i^{(1)}=0,\ Lv_i^{(1)}=a_i$$

where $\{a_1,\dots,a_m\}$ span $\mathbb{R}_m$.

By definition of g, see (2.4), it is clear that (3.1),(3.2) imply

(3.3) $$g=0,\ g_v=0.$$

We shall discuss consequences of the assumptions $g_\lambda=0$ and $g_{vv}=0$ and $g_{v\lambda}=0$.

Let us differentiate both (2.2) and (2.4) w.r.t. λ. It yields

$$Q[F_u w_\lambda+F_\lambda]=0,\ Lw_\lambda=0$$

and

$$g_\lambda=Q^c[F_u w_\lambda+F_\lambda].$$

Obviously, $g_\lambda=0$ if and only if

(3.4) $$\exists v_{m+1}^{(1)}\in U: F_u v_{m+1}^{(1)}+F_\lambda=0,\ Lv_{m+1}^{(1)}=0.$$

Namely,

(3.5) $$v_{m+1}^{(1)}=w_\lambda.$$

It follows from (2.4) that

$$g_{vv}=Q^c[F_{uu}\cdot(I+w_v)^2+F_u w_{vv}].$$

Let us calculate both w_v and w_{vv} from (2.2). Differentiating w.r.t. v,

$$Q[F_u\cdot(I+w_v)]=0,\ Lw_v=0.$$

Since $F_u v_i^{(1)}=0$ $(i=1,\dots,m)$,

(3.6) $$w_v v_i^{(1)}=0,\ i=1,\dots,m.$$

Differentiating (2.2) again,

$$Q[F_{uu}\cdot(I+w_v)^2+F_u w_{vv}]=0,\ Lw_{vv}=0.$$

It is simple to conclude that $g_{vv}=0$ if and only if $g_{vv}v_i^{(1)}v_j^{(1)}=0$ for $1\leq j\leq i\leq m$, which is equivalent to

(3.7) $$\begin{cases} \exists v_{ij}^{(2)}\in U\ (1\leq j\leq i\leq m): \\ F_u v_{ij}^{(2)}+F_{uu}v_i^{(1)}v_j^{(1)}=0,\ Lv_{ij}^{(2)}=0. \end{cases}$$

Namely,

(3.8) $$v_{ij}^{(2)}=w_{vv}v_i^{(1)}v_j^{(1)}.$$

Similar calculations yield the following assertion: $g_\lambda=0$, $g_{v\lambda}=0$ are equivalent to (3.4) and

(3.9) $$\begin{cases} \exists v_{m+1,j}^{(2)}\in U\ (j=1,\dots;m): \\ F_u v_{m+1,j}^{(2)}+F_{u\lambda}v_j^{(1)}+F_{uu}v_{m+1}^{(1)}v_j^{(1)}=0, \\ Lv_{m+1,j}^{(2)}=0 \end{cases}$$

with the interpretation

(3.10) $$v_{m+1,j}^{(2)}=w_{v\lambda}v_j^{(1)}\ (j=1,\dots,m).$$

We resume the above calculations in

Proposition 1. Assume Cases 1 - 3 of the definition G. Then the condition G=0 at $(v_0, \lambda_0, \alpha_0)$ is equivalent to the following conditions at $(u_0, \lambda_0, \alpha_0)$:

Case 1: (3.1),(3.2), (3.4);

Case 2: (3.1), (3.2), (3.7);

Case 3: (3.1), (3.2), (3.4), (3.7), (3.9).

The listed conditions define a root of an operator $\mathcal{F}$. In Case 1,

$$\mathcal{F}: U\times\mathbb{R}_1\times\mathbb{R}_k\times[U]^{m+1}\longrightarrow[Y]^{m+2}$$

is defined as follows: if $(u,\lambda,\alpha,v_1^{(1)},\dots,v_m^{(1)},v_{m+1}^{(1)})\in U\times\mathbb{R}_1\times\mathbb{R}_k\times[U]^{m+1}$ and $Lv_i^{(1)}=a_i$ for $i=1,\dots,m$ and $Lv_{m+1}^{(1)}=0$ then

$$(3.11)\quad \mathcal{F}(u,\lambda,\alpha,v_1^{(1)},\dots,v_{m+1}^{(1)})=\begin{pmatrix} F(u,\lambda,\alpha)\\ F_u(u,\lambda,\alpha)v_1^{(1)}\\ \vdots\\ F_u(u,\lambda,\alpha)v_m^{(1)}\\ F_u(u,\lambda,\alpha)v_{m+1}^{(1)}+F_\lambda(u,\lambda,\alpha)\end{pmatrix}$$

Thus $\mathcal{F}$ is defined on an affine subspace of $U\times\mathbb{R}_1\times\mathbb{R}_k\times[U]^{m+1}$. A simple shift of variables $v_i^{(1)}$ makes it possible to define $\mathcal{F}$ on the linear space $U\times\mathbb{R}_1\times\mathbb{R}_k\times[U_0]^{m+1}$, where

$$(3.12)\qquad U_0=\{u\in U: Lu=0\}.$$

A root $(u,\lambda,\alpha,v_1^{(1)},\dots,v_{m+1}^{(1)})$ has a clear interpretation: $(u,\lambda,\alpha)=x_0$ (i.e., it yields the singular point), the vectors $\{v_1^{(1)},\dots,v_m^{(1)}\}$ span $\mathrm{Ker}\,F_u(x_0)$ and $v_{m+1}^{(1)}=w_\lambda$.

The definition of $\mathcal{F}$ in Cases 2 and 3 is similar.

Remark. We have chosen comparatively simple examples of G. If, say, the condition G=0 includes the requirement that Hessian g_{vv} degenerates in one direction then a definition of $\mathcal{F}$ is not so straightforward. Nevertheless, we believe that any condition G=0 on an orbit of the germ $g(\cdot,\cdot,\alpha_0)$ centred at (v_0,λ_0) is equivalent to a condition $\mathcal{F}=0$ at $(u_0,\lambda_0,\alpha_0$, plus auxiliary variables) where $\mathcal{F}$ is the "inflated mapping" corresponding to F.

4. Gradient of the inflated mapping. Since the conditions G=0 at (v_0,λ_0,α_0) and $\mathcal{F}=0$ at $(u_0,\lambda_0,\alpha_0,\dots)$ are equivalent, one is ready to believe that the gradient DG at (v_0,λ_0,α_0) is invertible if and only if the gradient $D\mathcal{F}$ at $(u_0,\lambda_0,\alpha_0,\dots)$ is invertible. The invertibility of DG is formulated in the assumption (A), Section 2. We wish to discuss the statement: (A) holds if and only if $D\mathcal{F}$ is invertible at $(u_0,\lambda_0,\alpha_0,\dots)$.

We illustrate this statement on an example. Let us assume Case 1 of Section 3. The relevant $\mathcal{F}$ is defined by (3.11). Fréchet derivative $D\mathcal{F}$ at $(u_0,\lambda_0,\alpha_0,v_1^{(1)},\dots,v_{m+1}^{(1)})$ with respect to a direction $(\delta u,\delta\lambda,\delta\alpha,\delta v_1^{(1)},\dots,\delta v_{m+1}^{(1)})\in U\times\mathbb{R}_1\times\mathbb{R}_k\times[U_0]^{m+1}$ can be simply calculated:

$$(4.1)\quad D\mathcal{F}\,(\delta u,\delta\lambda,\delta\alpha,\delta v_1^{(1)},\dots,\delta v_{m+1}^{(1)})=(r,r_1^{(1)},\dots,r_{m+1}^{(1)})^T$$

where

$$(4.2)\qquad r=F_u\,\delta u+F_\lambda\,\delta\lambda+F_\alpha\,\delta\alpha$$

$$(4.3)\qquad r_i^{(1)}=(F_{uu}\,\delta u+F_{u\lambda}\delta\lambda+F_{u\alpha}\,\delta\alpha)v_i^{(1)}+F_u\,\delta v_i^{(1)}$$

for $i=1,\dots,m$, and

$$(4.4)\qquad \begin{cases} r_{m+1}^{(1)}=(F_{uu}\,\delta u+F_{u\lambda}\,\delta\lambda+F_{u\alpha}\,\delta\alpha)v_{m+1}^{(1)}+\\ \quad +F_{u\lambda}\delta u+F_{\lambda\lambda}\,\delta\lambda+F_{\lambda\alpha}\,\delta\alpha+F_u\,\delta v_{m+1}^{(1)}; \end{cases}$$

remind the convention that F (and its partials) are evaluated at $x_0=(u_0,\lambda_0,\alpha_0)$. We skip the argument $(u_0,\lambda_0,\alpha_0,v_1^{(1)},\dots,v_{m+1}^{(1)})$ of $\mathcal{F}$ and $D\mathcal{F}$, too.

Our aim is to prove that the linear mapping

$$D\mathcal{F}:U\times\mathbb{R}_1\times\mathbb{R}_k\times[U_0]^{m+1}\longrightarrow[Y]^{m+2}$$

is regular (i.e. it is invertible, with a bounded inverse).

Proposition 2. Assume Case 1 of Definition G. Let $(u_0,\lambda_0,\alpha_0,v_1^{(1)},\dots,v_{m+1}^{(1)})$ be a root of the relevant $\mathcal{F}$, see (3.11). Then the assumption (A) is equivalent to the statement that $D\mathcal{F}$, being evaluated at $(u_0,\lambda_0,\alpha_0,v_1^{(1)},\dots,v_{m+1}^{(1)})$, is regular.

Proof. By making use of formulas (4.1)-(4.4), we try to calculate the inverse of $D\mathcal{F}$. We use the notation

$$\delta v=\Pi\,\delta u,\qquad \delta w=\Pi^c\,\delta u;$$

i.e.

$$\delta u=\delta v+\delta w.$$

Projecting both sides of (4.2) by the operator Q onto the range of F_u, and making use of F_u^+ (see (2.1)), we calculate δw as an affine operator of $\delta\lambda$ and $\delta\alpha$. Namely,

$$(4.5)\qquad \delta w=w_\lambda\,\delta\lambda+w_\alpha\,\delta\alpha+R,\quad R=F_u^+r$$

where

$$(4.6)\qquad w_\lambda=-F_u^+F_\lambda,\quad w_\alpha=-F_u^+F_\alpha.$$

Projecting both sides of (4.2) by the projector Q^c, one can check that

$$(4.7)\qquad g_v\,\delta v+g_\lambda\,\delta\lambda+g_\alpha\,\delta\alpha=Q^cr.$$

Similarly, (4.3) and (4.5) imply

$$(4.8)\qquad \begin{cases} \delta v_i^{(1)}=(w_{vv}\,\delta v+w_{v\lambda}\,\delta\lambda+w_{v\alpha}\,\delta\alpha)v_i^{(1)}+\\ \qquad +R_i^{(1)}+w_{vv}v_i^{(1)}R,\ R_i^{(1)}=F_u^+r_i^{(1)} \end{cases}$$

where

$$(4.9)\qquad \begin{cases} w_{vv}=-F_u^+F_{uu},\ w_{v\lambda}=w_{vv}w_\lambda-F_u^+F_{u\lambda},\\ \qquad w_{v\alpha}=w_{vv}w_\alpha-F_u^+F_{u\alpha}. \end{cases}$$

Projecting (4.3) by Q^c, it yields

$$(4.10)\qquad (g_{vv}\,\delta v+g_{v\lambda}\,\delta\lambda+g_{v\alpha}\,\delta\alpha)v_i^{(1)}=Q^c[r_i^{(1)}-F_{uu}Rv_i^{(1)}].$$

Finally, as a consequence of (4.4), we obtain

$$(4.11)\qquad \begin{cases} \delta v_{m+1}^{(1)}=w_{v\lambda}\,\delta v+w_{\lambda\lambda}\,\delta\lambda+w_{\lambda\alpha}\,\delta\alpha+\\ \qquad +R_{m+1}^{(1)}+w_{v\lambda}R,\ R_{m+1}^{(1)}=F_u^+r_{m+1}^{(1)} \end{cases}$$

where

$$(4.12)\qquad \begin{cases} w_{\lambda\alpha}=w_{v\alpha}w_\lambda+w_{v\lambda}w_\alpha-w_{vv}w_\alpha w_\lambda-F_u^+F_{\lambda\alpha}\\ w_{\lambda\lambda}=2\,w_{v\lambda}w_\lambda-w_{vv}w_\lambda w_\lambda-F_u^+F_{\lambda\lambda}. \end{cases}$$

Moreover, (4.4) implies

$$(4.13)\qquad g_{v\lambda}\,\delta v+g_{\lambda\lambda}\,\delta\lambda+g_{\alpha\lambda}\,\delta\alpha=Q^c[r_{m+1}^{(1)}-(F_{uu}w_\lambda+F_{u\lambda})R].$$

Let us resume the above calculations. According to (4.5), (4.8) and (4.11), the vectors δw, $\delta v_i^{(1)}$ $(i=1,\dots,m)$, $\delta v_{m+1}^{(1)}$ are affine operators of $(\delta v,\delta\lambda,\delta\alpha)$. Continuity of these operators follows from the boundedness of F_u^+.

Denote $DG(\delta v,\delta\lambda,\delta\alpha)$ the Fréchet derivative of G at (v_0,λ_0,α_0) with respect to the direction $(\delta v,\delta\lambda,\delta\alpha)$. Then the conditions (4.7), (4.10) and (4.13) read as follows:

$$(4.14)\qquad DG(\delta v,\delta\lambda,\delta\alpha)=\begin{pmatrix} Q^c r\\ Q^c[r_1^{(1)}-F_{uu}Rv_1^{(1)}]\\ \vdots\\ Q^c[r_m^{(1)}-F_{uu}Rv_m^{(1)}]\\ Q^c[r_{m+1}^{(1)}-(F_{uu}w_\lambda+F_{u\lambda})R] \end{pmatrix},$$

where $R=F_u^+r$. Thus, $D\mathcal{T}$ is regular if and only if $(\delta v,\delta\lambda,\delta\alpha)$ depends continuously on $(r, r_1^{(1)}, \dots, r_m^{(1)})$ via (4.14).

We claim that the latter is equivalent to the assumption (A). For, note that G=0 counts $\ell = m(m+2)$ algebraic conditions. Identifying both Ker F_u and $Q^c Y$ with $\mathbb{R}_m$, the assumption (A) states that the linear operator

$$DG: \mathrm{Ker}\, F_u \times \mathbb{R}_1 \times \mathbb{R}_k \longrightarrow [Q^c Y]^{m+2}$$

is invertible.

5. Conclusions. The aim is to find a mapping $\mathcal{F}$ such that an organizing centre of F would be related to a simple root of $\mathcal{F}$. Our point is to link the construction of the mapping $\mathcal{F}$ with a classification of the organizing centre.

We have demonstrated this idea on three particular examples, see Proposition 1. The classification is not known a priori but it can be guessed using an auxiliary information (e.g. by means of codimension).

If the root of $\mathcal{F}$ is simple (for an example, see Proposition 2) then the Newton method can be applied to approximate the root.

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