ON MULTIVALUED AND SINGLEVALUED ACCRETIVE MAPPINGS

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Let X be a real normed linear space, X^* its dual. Recall that a mapping $A:X \longrightarrow 2^X$ is said to be:

(i) hemicontinuous (HC) at $u_0 \in \mathrm{int}_a D(A)$ (an algebraic interior of $D(A)$) if for any $v \in X$ and any null-sequence of positive numbers t_n and $x_n \in A(u_n)$, where $u_n = u_0 + t_n v \in D(A)$ for sufficiently large n, $x_n \longrightarrow x_0$ weakly in X and $x_0 \in A(u_0)$;

(ii) directionally upper semicontinuous (DUSC) at $u_0 \in \mathrm{int}_a D(A)$ if its restriction to any half line $L_v = \{u_0 + tu : t \geq 0\}$, $v \in X$ is upper semicontinuous (USC) at u_0;

(iii) demicontinuous (DC) at $u_0 \in D(A)$, if $(u_n) \subset D(A)$, $u_n \longrightarrow u_0$, $x_n \in A(u_n)$ imply that (x_n) converges weakly to x_0 and $x_0 \in A(u_0)$. Clearly, if A is (HC) at $u_0 \in \mathrm{int}_a D(A)$, then A is (DUSC) at u_0. Conversely, if A is singlevalued and (DUSC) at u_0, then A is (HC) at u_0. Similar relations are valid between (DC) and norm-to-weak (USC). The following results are related to that of [3] - [5].

Theorem 1. Let X be a reflexive Banach space, $A:X \longrightarrow 2^X$ an accretive mapping with $D(A) \subseteq X$. Then

(i) If X is smooth and rotund and A is singlevalued at $u_0 \in \mathrm{int}_a D(A)$, then A is (HC) at u_0;

(ii) If $\mathrm{int}\, D(A) \neq \emptyset$ and $A(u)$ is convex and bounded for each $u \in \mathrm{int}\, D(A)$ and the graph $G(A)$ of A is closed in $(X, \|\cdot\|) \times (X, \sigma(X,X^*))$, then A is singlevalued and (HC) on a dense G_δ subset of $\mathrm{int}\, D(A)$;

(iii) If X is Fréchet-smooth and A is (HC) at $u_0 \in \mathrm{int}\, D(A)$, then A is (DC) at u_0. Thm. 1(iii) extends the result of Kato [2], where it is assumed that X^* is uniformly rotund and A is singlevalued.

Theorem 2. Let X be a dual (i.e. $X = Z^*$ for some Banach space Z) smooth rotund and (H)-Banach space (i.e. if $(x_n) \subset X$, $x \in X$, $x_n \longrightarrow x$ weakly, $\|x_n\| \longrightarrow \|x\|$ imply that $x_n \longrightarrow x$), $A:X \longrightarrow 2^X$ a maximal accretive mapping with respect to the duality mapping $J:Z^* \longrightarrow Z$ and such that $D(A) \subset X$. If $\overline{R(A)}^{\sigma(Z^*,Z)}$ is convex, then $\lim_{\lambda \to +\infty} \frac{1}{\lambda} J_\lambda(u) = -a^0$ for each $u \in \bigcap_{\lambda > 0} (D(A) \cap R(I + \lambda A))$, where $J_\lambda = (I + \lambda A)^{-1}$ and a^0 is a unique element of $\overline{R(A)}^{\sigma(Z^*,Z)}$ with the minimum norm.

Using the result of [1] concerning the convexity of $\overline{R(A)}$ we get

Corollary 1. Let X be a reflexive rotund (H)-Banach space which is uniformly Gâteaux smooth (or equivalently X^* is weakly* uniformly rotund), $A:X \longrightarrow 2^X$ an m-accretive mapping with $D(A) \subset X$. Then $\lim_{\lambda\to+\infty} \frac{1}{\lambda} J_\lambda(u) = -a^0$ for each $u \in D(A)$, where a^0 is a unique point of $\overline{R(A)}$ with the minimum norm.

As a further consequence of Thm. 2 we obtain the result of [6] concerning maximal monotone mappings in Hilbert spaces.

References:

[1] L. GOBBO: On the asymptotic behavior of the resolvent of the inverse of an m-accretive operator in a Banach space, Rend.Sem.Mat.Univ.Politecn. Torino 42(1984), 47-64.

[2] T. KATO: Demicontinuity, hemicontinuity and monotonicity II, Bull.Amer.Math.Soc. 73(1967), 886-889.

[3] J. KOLOMÝ: Set-valued mappings and structure of Banach spaces, Rend.Circolo Mat.di Palermo (to appear).

[4] J. KOLOMÝ: Maximal monotone and accretive multivalued mappings and structure of Banach spaces, Proc.Int.Conference "Function Spaces", Poznań, August 25-29, 1986 (to appear).

[5] J. KOLOMÝ: On accretive multivalued mappings, Comment.Math. Univ.Carolinae 27(1986), 420.

[6] G. MOROSANU: Asymptotic behavior of resolvent of a monotone mapping in a Hilbert space, Atti Acad.Naz. dei Lincei 59(1976), 565-570.

MINIMAL CONVEX-VALUED WEAK USCO CORRESPONDENCES

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We say that a function $f:V \longrightarrow R$ defined on a vector space V is rotund if it is convex and $f((u+v)/2) < t$ whenever $u,v \in V$, $u \neq v$ and $f(u)=t=f(v)$. In what follows X will be a real Banach space.

Theorem 1. If there exists a weak* lower semicontinuous rotund function $f:X^* \longrightarrow R$, then X belongs to the Stegall class $\mathcal{S}$.

We denote by w* the weak* topology for any dual Banach space. Let D be a topological space. Then we write $F \in USCOC(F,(X^*,w^*))$ if and only if, using the weak* topology, F is a convex-valued usco correspondence from D into X^*. The set $USCOC(D,(X^*,w^*))$ is partially ordered with order $\leqq$, where $E \leqq F$ iff $E(d) \subset F(d)$ for each $d \in D$. We denote by $uscoc(D,(X^*,w^*))$ the set of all minimal elements of $USCOC(D,(X^*,w^*))$.

Theorem 2. Let $T:X \longrightarrow X^*$ be a maximal monotone operator and D be an open subset of X. If $Tx \neq \emptyset$ for all x in D then $T|D \in uscoc(D,(X^*,w^*))$.

If F is a correspondence from D into X^* then we define the set $C(F,D,X^*)$ as follows: $d \in C(F,D,X^*)$ if and only if $d \in D$ and, using the norm topology, F is upper semicontinuous and single-valued at d. In the following theorem X will be regarded as a closed vector subspace of X^{**}.

Theorem 3. Let K be a closed convex subset of X. Then K has the Radon-Nikodým property if and only if the set $C(F,D,X^{**})$ is dense in D whenever D is a Baire space, $F \in \mathrm{uscoc}(D,(X^{**},w^*))$ and the set $F^{-1}(K)$ is dense in D.

References:

[1] E. ASPLUND: Fréchet differentiability of convex functions, Acta Math. 121(1968), 31-47.

[2] P.R. CHRISTENSEN, P.S. KENDEROV: Dense strong continuity of mappings and the Radon-Nikodým property, Math. Scand. 54(1984), 70-78.

[3] S.P. FITZPATRICK: Monotone operators and dentability, Bull. Austr.Math.Soc. 18(1978), 77-82.

[4] P.S. KENDEROV: Multivalued monotone mappings are almost everywhere single-valued, Studia Math, LVI(1976), 199-203.

[5] P.S. KENDEROV: Monotone operators in Asplund spaces, C.R. Acad.Sci.Bulgare 30(1977), 963-964.

[6] C. STEGALL: More Gâteaux differentiability spaces, Banach Spaces, Proceedings, Missouri 1984.