ON THE REGULARITY OF WEAK SOLUTIONS TO NONLINEAR ELLIPTIC SYSTEMS OF PARTIAL DIFFERENTIAL EQUATIONS

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A regularity of weak solution for nonlinear elliptic system of partial differential equations is proved for the case of weak solution gradient of the system being unbounded and belonging to Campanato's space $\mathcal{L}^{2,n}$. Existing nonlinear system whose weak solution has its gradient in $\mathcal{L}^{2,n}$ space is also presented.

The considered nonlinear elliptic system whose gradient weak solution locally belongs to $\mathcal{L}^{2,n}$ space is in the form of

$$-D_i((a_{ij}^{rs}(x)D_j u_s) + g_i^r(x,u,Du)) + g^r(x,u,Du) = f^r(x),$$

where $r,s=1,\dots,N$, $i,j=1,\dots,n$, $N>1$, $n\geq 3$ are natural numbers, $x\in\Omega$, Ω is bounded, open subset in R^n and $u(x)\in W^{1,2}_{loc}(\Omega,R^N)$ is a weak solution of this system . The functions $a_{ij}^{rs}(x)$, $g_i^r(x,u,p)$, $g^r(x,u,p)$, $f^r(x)$ of the system fulfil certain hypotheses on the smooth and growth conditions in the variables $(u,p)\in R^N\times R^{nN}$.

The major part of the work is devoted to the proof of a new statement describing $C^{1,\alpha}$-regularity for the nonlinear elliptic system in divergent form

$$-D_i a_i^r(x,u,Du) + a^r(x,u,Du) = -D_i f_i^r(x) + f^r(x),$$

where $r = 1,\dots,N$, $N>1$, $x\in\Omega$, Ω bounded, open subset in R^n and $u(x)\in W^{1,2}_{loc}(\Omega,R^N)$ is a weak solution. The system is further assumed to fulfil Liouville's condition and the gradient of weak solution is assumed to belong to Campanato's space $\mathcal{L}^{2,n}$.

THE CENTRAL LIMIT PROBLEM FOR STRICTLY STATIONARY SEQUENCES

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Let $(X_i : i\in\mathbb{Z})$ be a strictly stationary sequence of random variables. Then there exists a function f and a bijective, bimeasurable and measure preserving transformation T on some probability space $(\Omega,\mathcal{A},\mu)$ such that $(X_i; i\in\mathbb{Z})$ have the same distributions. In this thesis the central limit problem is investigated for strictly stationary sequences $f\circ T^i; i\in\mathbb{Z}$) where $f\in L^2(\mu)$; more exactly, there is investigated the weak convergence of probability measures $\mu s_n^{-1}(f)$ where $s_n(f) = \frac{1}{\sqrt{n}}\sum_{j=1}^{n} f\circ T^j$. As a supporting but relatively independent result, a theory of decompositions of $(f\circ T^i; i\in\mathbb{Z})$ into sums of martingale difference sequences is developed.

A σ-algebra $\mathcal{M} \subset \mathcal{A}$ such that $\mathcal{M} \subset T^{-1}\mathcal{M}$, will be called invariant. If Λ is a linearly ordered set and for each $\alpha \in \Lambda$, $\mathcal{M}_\alpha$ is an invariant σ-algebra and for $\alpha < \beta$ it is $T^i \mathcal{M}_\alpha \supset T^j \mathcal{M}_\beta$ (for each $i,j \in \mathbb{Z}$), we say that ($\mathcal{M}_\alpha$; $\alpha \in \Lambda$) is a family of invariant σ-algebras. For an invariant σ-algebra $\mathcal{M}$, the projection operator onto $L^2(T^{-i-1}\mathcal{M}, \mu) \ominus L^2(T^{-i}\mathcal{M}, \mu)$ will be called a difference projection (generated by $\mathcal{M}$ in $L^2(\mu)$) and denoted by P_i. Given a family of invariant σ-algebras ($\mathcal{M}_\alpha$; $\alpha \in \Lambda$) we thus obtain a family of difference projections ($P_{-\infty}, P_{\alpha,i}$; $\alpha \in \Lambda$, $i \in \mathbb{Z}$) where $P_{-\infty}$ is the projection onto $L^2(\bigcap_{\alpha \in \Lambda} \bigcap_{i \in \mathbb{Z}} T^i \mathcal{M}_\alpha, \mu)$ and $P_{\alpha,i}$ are generated by $\mathcal{M}_\alpha$. If $f \in L^2(\mu)$ and there exists a family of difference projections such that $f = \sum_{\alpha \in \Lambda} \sum_{i \in \mathbb{Z}} P_{\alpha,i} f \mod \mu$, we shall say that f is difference decomposable. Note that for a difference projection operator P and $f \in L^2(\mu)$, $(Pf) \circ T^i$, $i \in \mathbb{Z}$ is a a martingale difference sequence.

Let $\mathcal{P}$ denote the Pinsker σ-algebra. If $f \in L^2(\mathcal{P}, \mu)$, we shall say that f is absolutely undecomposable.

Theorem 1. Let $\mathcal{Q} \subset \mathcal{P}$ be an invariant σ-algebra and Γ = ($\mathcal{M}_\alpha$; $\alpha \in \Lambda$) be a family of invariant σ-algebras. $\Gamma' = (\mathcal{M}_\alpha \vee \mathcal{Q}$; $\alpha \in \Lambda$) is then also a family of invariant σ-algebras. Let ($P_{-\infty}, P_{\alpha,i}$; $\alpha \in \Lambda, i \in \mathbb{Z}$) and ($P'_{-\infty}$, $P'_{\alpha,i}$; $\alpha \in \Lambda, i \in \mathbb{Z}$) be the families of difference projections generated by Γ , Γ'.
If $f \in L^2(\mu)$ and $f = \sum_{\alpha \in \Lambda} \sum_{i \in \mathbb{Z}} P_{\alpha,i} f + P_{-\infty} f \mod \mu$, then $P_{-\infty} f = P'_{-\infty} f \mod \mu$ and for each $\alpha \in \Lambda$, $i \in \mathbb{Z}$, it is $P_{\alpha,i} f = P'_{\alpha,i} f \mod \mu$.

Theorem 2. The set of all difference decomposable functions from $L^2(\mu)$ coincides with the Hilbert space $L^2(\mu) \ominus L^2(\mathcal{P}, \mu)$. For each difference decomposable function $f \in L^2(\mu)$ there exists an in variant σ-algebra $\mathcal{M}$ such that $f = \sum_{i \in \mathbb{Z}} P_i f \mod \mu$ (where P_i are generated by $\mathcal{M}$).

Let $\mathcal{J}$ be the σ-algebra of invariant sets from $\mathcal{A}$ (i.e. $A \in \mathcal{J}$ iff $A = TA$). Let us suppose that $\mathcal{A}$ has an at most countable generator and that there exist probability measures m_ω , $\omega \in \Omega$ such that for $A \in \mathcal{A}$, $\omega \mapsto m_\omega(A)$ is a version of the conditional probability $\mu(A \mathcal{J})$. For almost all $\omega \in \Omega[\mu]$, m_ω are ergodic measures and the following theorem holds.

Theorem 3. Let $\mathcal{M} \subset \mathcal{A}$ be an invariant σ-algebra and P_i, P_i^ω , $i \in \mathbb{Z}$, be the difference projections generated by $\mathcal{M}$ in $L^2(\mu)$, $L^2(m_\omega)$, $\omega \in \Omega$. Given $f \in L^2(\mu)$, for almost all $\omega \in \Omega[\mu]$, we have $P_i^\omega f = P_i f \mod m_\omega$.

Let $\mathcal{Z}$ denote the set of all functions $f \in L^2(\mu)$ such that there exists a family $(P_{-\infty}, P_{\alpha,i};\ \alpha \in \Lambda, i \in \mathbb{Z})$ and a natural number k such that $f = \sum_{\alpha \in \Lambda} \sum_{i=0}^{k} P_{\alpha,i} f \bmod \mu$. If $\inf_{g \in \mathcal{Z}} \limsup_{n \to \infty} \| s_n(f-g) \|_2 = 0$, we shall say that f is <u>finitely approximable.</u>

<u>Theorem 4</u>. If $f \in L^2(\mu)$ is a finitely approximable function, there exists a limit $\eta^2 = \lim_{n \to \infty} E(s_n^2(f)\ \mathcal{J})$ in $L^1(\mu)$ and the measures $\mu s_n^{-1}(f)$ weakly converge to a distribution with the characteristic function $\varphi : t \mapsto E \exp(-\frac{1}{2}\eta^2 t^2)$.
If $\eta^2 > 0$ a.s., then $\mu(\frac{1}{\eta} s_n(f))^{-1}$ weakly converge to the standard normal distribution.

<u>Theorem 5</u>. In the Hilbert space of difference decomposable functions, there exists a dense set of functions f such that $\mu s_n^{-1}(f)$ converge to some probability measure and provided there exists a nonzero difference decomposable function, the set of difference decomposable functions f such that the sequence $\mu s_n^{-1}(f)$, n=1,2,... has at least two different limit points, is dense (in the space of difference decomposable functions).

In deriving these results, several facts from ergodic theory (namely from entropic ergodic theory) and from martingale limit theory were used. I consider this thesis to be a continuation of the research made by M.I. Gordin, C.C. Heyde, G.K. Eagleson (preceded by the limit theorem of P. Billingsley and I.A. Ibragimov) and others. Several counterexamples to the CLP for strictly stationary processes (different from that one in Theorem 5) are given by R.C. Bradley.

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