THRESHOLD MOVING AVERAGE MODEL

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Define a partition of real line R ($R=S_1 \cup \dots \cup S_h$) by an ordered set of thresholds $\{p_1,\dots,p_{h-1}\}$; further define n real functions $b_k(.)$ constant on each of subsets S_i, $i=1,\dots,h$. Threshold moving average time series $\{X_t\}$ is defined by

$$X_t = Y_t + \sum_{k=1}^{m} b_k(Y_{t-u_k})\, Y_{t-k}, \quad t=\dots,-1,0,1,\dots ,$$

where $u_k \in N$ and $\{Y_t\}$ is the strict white noise.
The above model is studied from the point of view of stationarity, invertibility and estimation of parameters. Main results follow.

Stationarity. a) Let $u_k=k$, $k=1,\dots,n$. Then $\{X_t\}$ is stationary.

b) Let $u_k=d$, $d\in N$, $k=1,\dots,n$. Then $\{X_t\}$ is stationary. Mean and autocovariance function are calculated for the mentioned model; for n=1 and for the Gaussian white noise, the marginal density is obtained.

Invertibility. Let e_t be the error arisen from Granger-Andersen's procedure of estimation of white noise.

c) Let $h=2$; $p_1=0$; $|b_k(y)| \le \gamma_k$, $y\in R$, $k=1,\dots,n$ and $\sum \gamma_k < 1$. Then $\lim_{t\to\infty} E|e_t| = 0$.

d) Let $|b_k(y)| \le \gamma_k$, $y\in R$, $k=1,\dots,n$ and $\sum \gamma_k < 1$. Then there exists a real c such that $\limsup_{t\to\infty} E|e_t| < c$.

Estimation. For regular systems of densities, maximum likelihood estimators give consistent and asymptotically normal estimates of parameters.