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ON THE REGULARITY OF THE WEAK SOLUTION OF CAUCHY  
PROBLEM FOR NONLINEAR PARABOLIC SYSTEMS VIA  
LIOUVILLE PROPERTY  
O. JOHN, J. STARÁ

Dedicated to the memory of Svatopluk FUČÍK

**Abstract:** It is proved that Liouville property of initial value problem for parabolic quasilinear system - i.e. the fact that every bounded weak solution of the system with frozen coefficients and with zero initial data in  $R_+^{n+1}$  is zero - implies the  $C^{0,\alpha}$ -regularity of all bounded weak solutions of initial value problem up to the  $t=0$  part of the boundary. Moreover, if each bounded weak solution of a parabolic system is  $C^{0,\alpha}$ -regular, then Liouville property holds. Similar results for interior parabolic regularity were proved in [12], for elliptic systems in [5],[6],[7],[8],[9],[10],[11].

**Key words:** Quasilinear parabolic system, initial value problem, regularity up to the boundary, parabolic Liouville property.

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**Introduction.** It is well known that the bounded weak solution of a quasilinear parabolic system need not be Hölder-continuous. In [12] there was proved that Hölder-continuity of a solution in the interior of the domain is guaranteed if for the system in question certain Liouville type theorem (see Definition 4) holds.

We shall prove here that Hölder-continuity up to the part of the boundary contained in the hyperplane  $t=0$  is a consequence of a similar Liouville type theorem for solutions on

halfspace with constant initial data (see Definition 5) under the assumption that the initial data are sufficiently smooth.

There is a counterexample due to M. Struwe (see [4]) showing that a bounded weak solution starting from the smooth initial data can develop a singularity. However, in this counterexample the parabolic system does not satisfy the conditions imposed here because of the quadratic dependence of the right hand side on the gradient of the solution.

We are deeply indebted to M. Struwe for fruitful discussions.

I. Notations and definitions. Let  $\Omega$  be a domain in  $\mathbb{R}^n$ . Denote for a  $T_0 \in (0, \infty)$

$$Q^+ = \{z=(t,x) \in \mathbb{R}^{1+n}; t \in (0, T_0), x \in \Omega\},$$

$$\Gamma = \{z=(0,x) \in \mathbb{R}^{1+n}; x \in \Omega\},$$

$$Q^- = \{z=(t,x) \in \mathbb{R}^{1+n}; (-t,x) \in Q^+\}$$

and

$$Q = Q^+ \cup \Gamma \cup Q^-.$$

By  $L_p(Q)$ ,  $W_p^k(Q)$ ,  $C^{0,\alpha,\alpha/2}(Q)$  will be denoted the corresponding Lebesgue and Sobolev spaces and the spaces of Hölder-continuous functions.

Let the nonlinear parabolic system in the form

$$\frac{\partial u^i}{\partial t} - \frac{\partial}{\partial x_\alpha} (a_{ij}^{\alpha\beta}(z;u) \frac{\partial u^j}{\partial x_\beta}) = -f^i(z) + \frac{\partial}{\partial x_\alpha} g_1^\alpha(z),$$

$$i, j=1, \dots, m; \quad \alpha, \beta=1, \dots, n$$

be given. For the sake of simplicity we rewrite it in the matrix form

$$(1) \quad u_t - \operatorname{div}_x (A(z;u) D_x u) = -f(z) + \operatorname{div}_x g(z).$$

First, we introduce the concept of the weak solution of both the system (1) and of the Cauchy problem for this system. (Functions  $A, f, g, u_0$  are supposed to be defined on the corresponding sets.)

Definition 1. The function  $u \in W_{2,loc}^{0,1}(Q) \cap L_\infty(Q)$  is said to be a weak solution of the system (1) in  $Q$  if

$$(2) \quad \forall \varphi \in C_0^\infty(Q): \int_Q [u \varphi_t - A(z;u) D_x u D_x \varphi] dz = \int_Q [f \varphi + g D_x \varphi] dz.$$

Definition 2. The function  $u \in W_{2,loc}^{0,1}(Q^+ \cup \Gamma) \cap L_\infty(Q^+)$  is called a weak solution of the Cauchy problem for the system (1) in  $Q^+$  with the initial value  $u_0$  if

$$(3) \quad \forall \varphi \in C^\infty(\overline{Q^+}), \text{ supp } \varphi \subset Q^+ \cup \Gamma$$

$$\int_{Q^+} [u \varphi_t - A(z;u) D_x u D_x \varphi] dz = \int_{Q^+} [f \varphi + g D_x \varphi] dz - \int_\Gamma u_0(x) \varphi(0,x) dx.$$

In what follows, the functions  $A, f, g, u_0$  satisfy the conditions

- (4)  $A(z;p)$  is continuous on  $(Q^+ \cup \Gamma) \times \mathbb{R}^m$ ;
- (5)  $(A(z;p) \xi, \xi) > 0$  for all  $(z;p) \in (Q^+ \cup \Gamma) \times \mathbb{R}^m$ ,  $\xi \neq 0$ ;
- (6)  $f \in L_{s,loc}(Q^+ \cup \Gamma)$  with  $s > n/2 + 1$ ;
- (7)  $g \in L_{q,loc}(Q^+ \cup \Gamma)$  with  $q > n + 2$ ;
- (8)  $u_0 \in W_{r,loc}^1(\Gamma) \cap L_\infty(\Gamma)$  with  $r > n$ .

Now, the properties (Li), (Lb) of the Liouville type are defined. They concern the behaviour of a weak solution of (1) in the whole space  $\mathbb{R}^{1+n}$  (resp. of a weak solution of the Cauchy problem for (1) in  $\mathbb{R}_+^{1+n}$ ) in case  $f=0, g=0, u_0=0$  and  $A$  being frozen in an arbitrary point  $z_0 \in Q^+$  (resp.  $z_0 \in \Gamma$ ).

Definition 3 (Li). We shall say that the system (1) has Liouville property (Li) if the following assertion holds:

Let  $z_0$  be a generic point of  $Q^+$  and let the function  $u$  be a weak solution of the system

$$(9) \quad u_t - \operatorname{div}_x(A(z_0; u) D_x u) = 0$$

on  $\mathbb{R}^{1+n}$ . Then  $u$  is constant.

**Definition 4** (Lb). We shall say that the system (1) has Liouville property (Lb) if the following assertion holds:

Let  $z_0$  be an arbitrary point of  $\Gamma$ . Let  $u$  be a weak solution of the Cauchy problem for the system (9) in  $\mathbb{R}_+^{1+n}$  with the initial value  $u_0=0$ . Then  $u$  is zero.

We should like to prove that each system (1) satisfying both (Li) and (Lb) is regular in the following way:

**Definition 5** (Re). Let  $u$  be a weak solution of the Cauchy problem for (1) with the initial value  $u_0$  satisfying (8). Then there exists  $\alpha \in (0,1)$  such that  $u \in C_{loc}^{0, \alpha/2, \alpha}(Q^+ \cup \Gamma)$ .

**Remark.** Cauchy problem for (1), being regular in the sense of Definition 5, is regular with the maximal exponent corresponding to the regularity of  $u_0$  and right hand side. It can be proved in the following way:

The function  $u \in C_{loc}^{0, \alpha/2, \alpha}$  substituted to  $A(z; u)$  in (1) enables us to treat it as a linear system with Hölder-continuous coefficients. Applying Schauder estimates we obtain that the maximal coefficient  $\alpha_1$  of hölderianity of the solution  $u$  is determined by the quality of  $f$ ,  $g$  and  $u_0$ .

## II. Main theorem

**Theorem.** Let the system (1) have the properties (Li) and (Lb). Then it has the property (Re).

Sketch of the proof. We extend the coefficients and the right hand side functions of (1) to the whole cylinder  $Q$ . After that the weak solution  $u$  of the Cauchy problem for (1) in  $Q^+$  can be shifted and prolonged in a suitable manner to the weak solution  $w$  of the extended system on the whole  $Q$ . The (Li) and (Lb) imply that  $w$  is in  $C_{loc}^{0,\alpha/2,\alpha}(Q)$  with an  $\alpha \in (0,1)$ . Thus the assertion of the Theorem follows immediately.

Proof. Let  $u$  be a weak solution of the Cauchy problem for (1) with the initial condition  $u_0$ . Put

$$(10) \quad v(z) = u(z) - u_0(x).$$

Substituting to (3) we check immediately that  $v$  satisfies the integral identity

$$(11) \quad \forall \varphi \in C^\infty(\overline{Q^+}), \text{supp } \varphi \subset Q^+ \cup \Gamma$$

$$\int_{Q^+} [v \varphi_t - A(z; v+u_0) D_x v D_x \varphi] dz = \int_{Q^+} [f \varphi + G D_x \varphi] dz,$$

where

$$(12) \quad G(z) = g(z) - A(z; v(z) + u_0(x)) D_x u_0(x).$$

Denote for  $z_0 = (t_0, x_0) \in \mathbb{R}^{1+n}$ ,  $R > 0$

$$(13) \quad Q(z_0, R) = \{z = (t, x); t \in (t_0 - R^2, t_0), |x - x_0| < R\} = \\ = (t_0 - R^2, t_0) \times B(x_0, R).$$

In the next lemma we show that the function  $G$  has the quality needed in what follows.

Lemma 1. Let the assumptions (4) - (8) hold. Then for each  $b > 0$ ,  $M > 0$  there exists  $C > 0$  such that for each  $Q(z_0, R) \subset \subset Q^+ \cup \Gamma$  with  $z_0 \in Q^+$ ,  $R < 1$ ,  $\text{dist}(Q(z_0, R), \partial Q^+ \setminus \Gamma) > b$  and for each  $v \in L_\infty(Q^+)$ ,  $\|v\|_\infty < M$  it is

$$(14) \quad R^{-(n+\lambda)} \int_{Q(z_0, R)} |g(z)|^2 dz \leq 0$$

where

$$(15) \quad \lambda = \min \left\{ \frac{2}{q} [q - (n+2)], \frac{2}{r} (r-n) \right\} > 0.$$

To prove it, we use the assumptions (4) - (8) and Hölder inequality.

Now, we extend the system (1) to the whole domain  $Q$ . Put

$$(16) \quad \begin{aligned} A_{\bullet}(z;p) &= \begin{cases} A(z;p+u_0(x)), & z \in Q^+, \\ A((0;x);p+u_0(x)), & z \in Q^-, \end{cases} \\ f_{\bullet}(z) &= \begin{cases} f(z), & z \in Q^+, \\ 0, & z \in Q^-, \end{cases} \\ G_{\bullet}(z) &= \begin{cases} G(z), & z \in Q^+, \\ 0, & z \in Q^-. \end{cases} \end{aligned}$$

It can be easily verified that

$$(17) \quad A_{\bullet}(z;p) \text{ is continuous on } Q \times \mathbb{R}^m,$$

$$(18) \quad (A_{\bullet}(z;p) \xi, \xi) > 0 \text{ for all } (z;p) \in Q \times \mathbb{R}^m, \quad \xi \neq 0,$$

$$(19) \quad f_{\bullet} \in L_{s,loc}(Q) \text{ with the same } s \text{ as for } f,$$

$$(20) \quad \text{the assertion of Lemma 1 remains valid for the function } G_{\bullet} \text{ and } Q(z_0, R) \subset \subset Q.$$

We formulate the next obvious result as

Lemma 2. The function

$$(21) \quad v_{\bullet}(z) = \begin{cases} v(z) & \text{on } Q^+ \\ 0 & \text{on } Q^- \end{cases}$$

is a weak solution of the system

$$(22) \quad w_t - \operatorname{div}_x (A_{\bullet} D_x w) = -f_{\bullet} + \operatorname{div}_x G_{\bullet}$$

on  $Q$ .

Denote further

$$(23) \quad h_{z_0, R} = \frac{1}{\mu Q(z_0, R)} \int_{Q(z_0, R)} h(z) \, dz,$$

$$(24) \quad \int_{Q(z_0, R)} |h(z)|^2 \, dz = R^{-n-2} \int_{Q(z_0, R)} |h(z)|^2 \, dz.$$

Definition 6. Let  $w$  be a weak solution of (21) in  $Q$ . A point  $z_0 \in Q$  is said to be a regular point of  $w$  if

$$(25) \quad \lim_{R \rightarrow 0} \int_{Q(z_0, R)} |w(z) - w_{z_0, R}|^2 \, dz = 0.$$

Lemma 3. Each point of  $Q$  is a regular point of the weak solution  $v_e$  of the system (22).

Proof. Let  $z_0 = (t_0, x_0) \in Q$  be fixed,  $Q(z_0, R) \subset \subset Q$ . To prove that  $z_0$  is regular we substitute first

$$(26) \quad T = (t - t_0)R^{-2}, \quad X = (x - x_0)R^{-1}, \quad Z = (T, X), \\ v_R(T, X) = v_e(t_0 + R^2 T, x_0 + R X).$$

For an arbitrary constant vector  $H \in \mathbb{R}^m$  we get

$$(27) \quad \int_{Q(z_0, R)} |v_e(z) - (v_e)_{z_0, R}|^2 \, dz \leq \int_{Q(z_0, R)} |v_e(z) - H|^2 \, dz = \\ = \int_{Q(0, 1)} |v_R(Z) - H|^2 \, dZ.$$

(The first inequality in (27) is due to the fact that the functional

$$I(H) = \int_{Q(z_0, R)} |w(z) - H|^2 \, dz$$

attains its minimum on  $\mathbb{R}^m$  in the point  $H = w_{z_0, R}$ .)

Thus,  $z_0$  is a regular point of  $v_e$  if there exists a sequence  $\{v_{R_n}\}$  ( $R_n \rightarrow 0 +$  as  $n \rightarrow \infty$ ) such that

$$(28) \quad v_{R_n} \rightarrow p \text{ in } L_2(Q(0, 1)),$$



(29)  $p$  is a constant vector function.

To prove (28) and (29) we go back to the system (22), substituting there for  $t, x, v_e$  from (26) and using the notation

$$(30) \quad \begin{aligned} A_R(Z) &= A_e(t_0 + TR^2, x_0 + XR, v_R(Z)) \\ f_R(Z) &= f_e(t_0 + TR^2, x_0 + XR), \\ G_R(Z) &= G_e(t_0 + TR^2, x_0 + XR), \end{aligned}$$

we see that  $v_R(Z)$  weakly solves the system

$$(31) \quad (w)_T - \operatorname{div}_X (A_R(Z) D_X w) = -f_R + \operatorname{div}_X G_R \quad \text{on } (Q)_R,$$

where  $(Q)_R$  is the image of  $Q$  in the mapping (26).

$R > 0$  going to zero, the set  $(Q)_R$  expands to the whole space  $\mathbb{R}^{1+n}$ . Thus, choosing  $K > 0$ , we obtain that

$$(32) \quad \exists R(K) > 0: Q(0, K) \subset C(Q)_R \text{ for all } R < R(K).$$

It follows that each  $v_R$  ( $R < R(K)$ ) is the solution of the system

$$(33) \quad \forall \varphi \in C_0^\infty(Q(0, K))$$

$$\int_{Q(0, K)} [v_R \varphi_T - A_R(Z) D_X v_R D_X \varphi] dz = \int_{Q(0, K)} [R^2 f_R \varphi + R G_R D_X \varphi] dz$$

The class of systems (33) can be interpreted as a class of linear parabolic systems with bounded measurable coefficients  $\{A_R\}_{R < R(K)}$ . Because of the estimate

$$(34) \quad \|v_R\|_{L_\infty(Q(0, K))} \leq \|v_e\|_{L_\infty(Q)}$$

and the continuity of  $A_e(z, p)$  we can deduce that the coefficients  $A_R$ ,  $R < R(K)$ , are equibounded and that all the systems of the class have the same constant  $\gamma$  of ellipticity.

To prove that  $\{v_R\}_{R < R(K)}$  is a compact set in  $L_2(Q(0, K/2))$

we take use of the Caccioppoli type estimate (see [1], (3.1)). Taking account of the possibility to estimate  $L_2$ -norm of  $D_X v_R$  by means of the  $L_2$ -norm of  $v_R$  itself (see e.g. [3], Lemma 2.1) over the larger domain, we get finally

$$(35) \quad \|v_R\|_{W_2^{1/2,1}(Q(0,K/2))}^2 \leq c(1 + \|v_R\|_{L_2(Q(0,K))}^2), \quad R < R(K).$$

From (34) and (35) follows

$$(36) \quad \|v_R\|_{W_2^{1/2,1}(Q(0,K/2))}^2 \leq c(K), \quad R < R(K).$$

Because of the compactness of the imbedding of  $W_2^{1/2,1}$  into  $L_2$  it follows from (36) that we can choose the sequence  $\{v_k\} = \{v_{R_k}\}$ ,  $\lim_{k \rightarrow \infty} R_k = 0$ , for which

$$(37) \quad \begin{aligned} \{v_k\} &\text{ converges to a function } p \text{ in } L_2(Q(0,K/2)), \\ D_X v_k &\rightarrow D_X p \text{ in } L_2(Q(0,K/2)), \\ v_k &\rightarrow p \text{ almost everywhere in } Q(0,K/2). \end{aligned}$$

By means of the diagonal method we get the subsequence of  $\{v_k\}$  (keeping the same notation for it) such that for each bounded domain  $D \subset \mathbb{R}^{1+n}$  it is

$$(38) \quad \begin{aligned} v_k &\rightarrow p \text{ and } D_X v_k \rightarrow D_X p \text{ in } L_2(D), \\ v_k &\rightarrow p \text{ almost everywhere on } \mathbb{R}^{1+n}, \\ &\text{(in particular } p \in L_\infty(\mathbb{R}^{1+n})). \end{aligned}$$

Assumptions (6) - (8) and Lemma 1 give

$$(39) \quad R_k^2 f_k \rightarrow 0 \text{ and } R_k G_k \rightarrow 0 \text{ in } L_2(D).$$

( $f_k = f_{R_k}$  and for the definition of  $f_R$  see (30); similarly for  $G_k$ .)

Let now  $\varphi$  be a fixed function of  $C^\infty(\mathbb{R}^{1+n})$  with a compact support. We can rewrite (33) as

$$(40) \int_{\mathbb{R}^{1+n}} [v_k \varphi_T - A_k(Z) D_X v_k D_X \varphi] dZ = \int_{\mathbb{R}^{1+n}} [R_k^2 f_k \varphi + R_k G_k D_X \varphi] dZ.$$

According to (39), the right hand side of (40) tends to zero. Thanks to the uniform boundedness of the set  $\{A_k\}$  on  $\text{supp } \varphi$  and the almost everywhere convergence

$$(41) \quad \lim_{k \rightarrow \infty} A_k(Z) = A_\theta(z_0, p(Z)),$$

we get that the vector function  $p$  solves the equation

$$(42) \int_{\mathbb{R}^{1+n}} [p \varphi_T - A_\theta(z_0; p) D_X p D_X \varphi] dZ = 0, \quad \forall \varphi \in C^\infty(\mathbb{R}^{1+n})$$

supp  $\varphi$  is compact.

If  $z_0 \in Q^-$ , then (25) with  $w = v_\theta$  is trivial and  $z_0$  is a regular point of  $v_\theta$ .

If  $z_0 \in Q^+$ , then (42) means that the vector function  $p$  is the weak solution of the system

$$(43) \quad p_t - \text{div}_X (A(z_0; p + u_\theta(x_0)) D_X p) = 0$$

in  $\mathbb{R}^{1+n}$ . According to (Li),  $p$  is a constant vector function and thus  $z_0$  is regular, too.

If, finally,  $z_0 \in \Gamma$ , then (42) gives that the  $p$  is a weak solution of the Cauchy problem with zero initial value for the system (43). So  $p \equiv 0$  on  $\mathbb{R}_+^{1+n}$ , according to (Lb). From the trivial fact that  $p \equiv 0$  on  $\mathbb{R}_-^{1+n}$ , we have again that  $z_0$  is a regular point of the solution  $v_\theta$  of (22) and the proof of Lemma 3 is completed.

As it was proved in [1],[2],[3], if for a weak solution of (22) all points of  $Q$  are regular, then  $v_\theta \in C_{\text{loc}}^{0, \alpha/2, \alpha}(Q)$  and thanks to the assumptions on  $u_\theta$ ,  $u \in C_{\text{loc}}^{0, \alpha/2, \alpha}(Q^+ \cup \Gamma)$ .

Remark. Let us mention now the "almost necessity" of Liouville condition:

Let  $z_0 \in \Gamma$ , let the system

$$(44) \quad u_t - \operatorname{div}_x (\Lambda(z_0, u) D_x u) = 0$$

have the property (Re). Let  $u$  be a weak solution of the Cauchy problem for (44) on  $\mathbb{R}_+^{1+n}$  with zero initial data. Let  $z$  be an arbitrary point of  $\mathbb{R}_+^{1+n}$ . We shall prove that (Re) implies that  $u(z) = u(0) = 0$ .

Let  $Q^+$  be a set described in Sec. I which is, in addition, convex, bounded and such that the points 0 and  $z$  are contained in  $\overline{Q^+}$ . Using (Re) we get the existence of a constant  $C$  such that for every solution  $v$  of (44) with zero initial data the estimate

$$(45) \quad \|v\|_{C^{0, \alpha/2, \alpha}(\overline{Q^+})} \leq c(\|v\|_{L_\infty(\mathbb{R}_+^{1+n})})$$

holds.

Putting  $u_R(t, x) = u(tR^2, xR)$  we get a sequence of solutions of (44) with zero initial data and the same bound for  $\|u_R\|_{L_\infty}$ . Thus for all  $R \geq 1$  the norms

$$\|u_R\|_{C^{0, \alpha/2, \alpha}(\overline{Q^+})}$$

are equibounded. Let  $R \geq 1$ ,  $z = (t, x) = (tR^2, xR)$ . Then  $(t, x) \in \overline{Q^+}$  and

$$\begin{aligned} |u(z) - u(0)| &= |u_R(t, x) - u_R(0)| = c(|x|^\alpha + |t|^{\alpha/2}) = \\ &= cR^{-\alpha}(|x|^\alpha + |t|^{\alpha/2}). \end{aligned}$$

Letting  $R \rightarrow \infty$  we obtain  $u(z) = u(0)$ .

So we proved that the condition (Re) for the system (44) yields (Lb) in the point  $z_0$ . Similar assertion can be proved in the interior point  $z_0$ .

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