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ADEQUATE FAMILIES OF SETS AND CORSON COMPACTS

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Abstract: In this paper we construct an example of a Corson compact X for which the space $C_p(X)$ fails to be a Lindelöf Σ-space. This example gives the negative answer for one problem of A.V. Arhangel'skiĭ. The notion of an adequate family is used. We establish its connection with the classes of Eberlein and Corson compacts and also with some set theoretic problems.

Key words and phrases: Corson compact, Eberlein compact, adequate family of sets, partially ordered set, Lindelöf Σ-space.

Classification: 54C40

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1. Introduction. The main result of this paper is the final solution of the problem of A.V. Arhangel'skiĭ [1]: are the following conditions

(1) X is a Corson compact;

(2) The space $C_p(X)$ is a Lindelöf Σ-space;

equivalent for a compact space X ?

The most general results concerning the Lindelöf property of the space $C_p(X)$ were obtained by K. Alster, R. Pol [5] and S.P. Gul'ko [2] who proved that $C_p(X)^{\aleph_0}$ is Lindelöf for every Corson compact X. R. Pol [6] constructed an example of a compact space X with the properties that $C_p(X)$ is Lindelöf and X is not a Corson compact.

In accordance with M. Talagrand [7], we denote by $\mathcal{E}_1$ and $\mathcal{E}_2$ the classes of all compact spaces X for which $C_p(X)$ is $\mathcal{K}$-analytic and a Lindelöf Σ-space respectively.

It is worth while mentioning that the class $\mathcal{E}_2$ exactly consists of the compact spaces X, the Banach space C(X) of which L. Vašák [4] calls WCD.

For classes of Eberlein and Corson compacts we use the symbols $\mathcal{E}$ and $\mathcal{K}$ respectively.

M. Talagrand [8] proved $\mathcal{E} \subset \mathcal{E}_1$ and showed in [9] that these classes are strictly different. It is well known that $\mathcal{E}_1 \subset \mathcal{E}_2$, but the question about the coincidence of these classes is still open. K. Alster and R. Pol [5] constructed an example showing that $\mathcal{E}_1 \neq \mathcal{K}$. The inclusion $\mathcal{E}_2 \subset \mathcal{K}$ (i.e. implication (2) $\Longrightarrow$ (1) was proved by S.P. Gul'ko [3]. Notice that the same conclusion easily follows from the L. Vašák's work [4].

In this paper we show that the converse inclusion (i.e. implication (1) $\Longrightarrow$ (2)) does not hold. The notion of an adequate family of sets is essentially used throughout the paper. The definition of bushes is given as a natural generalization of trees. We construct once more an example of an Eberlein compact which is not a uniform Eberlein compact. This example is much simpler than the analogous one of Y. Benyamini and T. Starbird [10].

All the results with the exception of Example 5.2 are obtained by the first author.

2. Terminology and notation. Our terminology is standard. The symbol N stands for the set of natural numbers; $\mathbb{R}$

is the real line; $|T|$ denotes the cardinality of a set T; ω_1 is the first uncountable ordinal; $\mathcal{D} = \{0,1\}$ stands for the two-point discrete space.

For a compact space X we denote by $C_p(X)$ the space of all real-valued functions on X endowed with the pointwise topology.

For a topological space X let $d(X)$ be the density of X and $c(X)$ be the Souslin number of X. The closure of a subset $A \subset X$ is denoted by $[A]_X$.

Recall that Corson, Eberlein, strong Eberlein and uniform Eberlein compacts are the compact subspaces of

$$\Sigma(\mathbb{R},T) = \{x \in \mathbb{R}^T : |\operatorname{supp} x| \leq \aleph_0\},$$

where $\operatorname{supp} x = \{t \in T : x(t) \neq 0\}$;

$$c_0(\mathbb{R},T) = \{x \in \mathbb{R}^T : |\{t \in T : |x(t)| > \varepsilon\}| < \aleph_0 \ \forall \varepsilon > 0\};$$

$$\sigma(\mathcal{D},T) = \{x \in \mathcal{D}^T : |\operatorname{supp} x| < \aleph_0\};$$

$$\ell_2(\mathbb{R},T) = \{x \in \mathbb{R}^T : \sum_{t \in T} |x(t)|^2 < \infty\},$$

respectively.

A completely regular space Z is a Lindelöf Σ-space if there is a countable collection of closed subsets $\{F_n\}_{n \in N}$ such that for each $z \in Z$ the set $B_z = \bigcap \{[F_n]_{\beta Z} : z \in F_n\}$ is nonempty and contained in Z, where βZ is the Stone-Čech compactification of Z. We can assume that the collection $\{F_n\}_{n \in N}$ is closed under finite intersections, therefore, if U is any neighborhood of B_z in Z then $B_z \subset F_n \subset U$ for some $n \in N$.

If $(T, \leq)$ is a partially ordered set, then $p, q \in T$ are compatible if there exists $s \in T$ such that $s \leq p$, $s \leq q$, otherwise p and q are incompatible. $(T, \leq)$ is ccc if T does not contain an uncountable subset of pairwise incompatible elements. Elements $p, q \in T$ are comparable if $p < q$ or $q \leq p$ holds, otherwise p and q are incomparable. Every totally ordered subset of $(T, \leq)$ is called a chain.

3. Construction. The following definition introduced in [7] plays the key role.

Definition 3.1. Let T be a set. A family $\mathcal{A}$ of its subsets is called an adequate (n-adequate) if it satisfies the following conditions:

i) $\mathcal{A}$ contains all one-point subsets of T.

ii) A subset A of T belongs to $\mathcal{A}$ iff every finite (k-point, $k \leq n$) subset of A belongs to $\mathcal{A}$.

It follows from the definition of $\mathcal{A}$ that if $A \in \mathcal{A}$, $B \subset A$, then $B \in \mathcal{A}$. Put $X = X_{\mathcal{A}} = \{\chi_A ; A \in \mathcal{A}\} \subset \mathfrak{D}^T$, where χ_A is the characteristic function of A. As observed in [7], if $\mathcal{A}$ is an adequate family, then X is a compact space. We call X an adequate compact in this case. Evidently, X is the Corson compact if $\mathcal{A}$ consists of at most countable sets.

The above constructed compact space on the 2-adequate family of sets coincides exactly with "the space of complete subgraphs of a graph" defined by M. Bell [11].

The property to be a remainder of the countable discrete space which he investigates is apart from the subject of our paper.

A family of all chains of an arbitrary partially ordered set is the most useful example of adequate families.

Definition 3.2. A partially ordered set $(T, \leq)$ is called a bush if for every $t \in T$ the set $\hat{t} = \{s \in T : s < t\}$ is totally ordered. A bush is called an A-bush if it does not contain an uncountable chain. A pairwise incomparable subset of a bush is called an antichain. Finally, an A-bush is an S-bush provided $|T| = \aleph_1$ and it does not contain an uncountable antichain.

The notion of a bush naturally generalizes the known concept of a tree which we should obtain if we demand that the sets $\hat{t}$ are well ordered. In this case, under the additional assumptions that all levels are nonempty and countable, any A-bush is an Aronszajn tree and any S-bush is a Souslin tree. An Aronszajn tree which is a union of a countable family of antichains is called special [13].

4. Results. Henceforth, $X = X_{\mathcal{A}}$ is an adequate compact; $\mathcal{A}$ is an adequate family of subsets of T. Consider the subset of $C_p(X)$ $\{\delta_t : t \in T\} \cup \{\mathbf{0}\}$, where $\delta_t(x) = x(t)$, $x \in X$ and $\mathbf{0}$ is the constant zero-valued function. It is known [7] that this set is closed in $C_p(X)$ and is homeomorphic with the space $T^* = T \cup \{*\}$ endowed with the following topology: T is the discrete subspace of T^* and every neighborhood of the point $\{*\}$ is the complement of finite unions of members of $\mathcal{A}$.

The fact that T^* is closed in $C_p(X)$ and separates the points of X yields

Proposition 4.1. [7]. The space $C_p(X)$ is a Lindelöf Σ-space if and only if T^* is the same.

Theorem 4.2. Let $(T, \leq)$ be a bush. Let $\mathcal{A}$ be a family of its chains and $X = X_{\mathcal{A}}$ be an adequate compact. Then $C_p(X)$ is a Lindelöf Σ-space if and only if T is a union of a countable family of antichains.

Proof: (if). Assume that $T = \bigcup_{n \in N} T_n$, where every T_n is an antichain. Then $T_n \cup \{*\}$ is the one-point compactification of the discrete space T_n for every $n \in N$, hence, T^* has the type K_σ. Consequently, in this case $C_p(X)$ has the type $K_{\sigma\delta}$ (cf.[7])

and, moreover, it is a Lindelöf Σ-space.

(only if). If $C_p(X)$ is a Lindelöf Σ-space then, according to Proposition 4.1, the space T^* is the same. By the definition, there is a sequence of sets $\{F_n\}_{n\in N}$ and a collection of compacts $B_t \ni t$ for each point $t\in T$ so that, for every neighborhood U of the set B_t, there is $n\in N$ such that $B_t \subset F_n \subset U$. Denote by $A_t = T^* \setminus (\hat{t} \setminus B_t)$, $t \in T$ and $V_n = \{t\in T: t \in F_n \subset A_t\}$, $n \in N$. The set A_t is open and contains B_t, hence, it is clear that the family $\{V_n\}_{n\in N}$ covers T. Observe that the compact B_t does not contain an infinite discrete subset, therefore, it follows from the definition of the topology on T^* that for each $t\in T$ the set B_t does not contain an infinite chain. From this we conclude that the set $\hat{t} \cap V_n$ is finite, because $V_n \subset F_n \subset A_t$ and $\hat{t}\cap V_n \subset \hat{t} \cap B_t$ for every $t\in V_n$. Denote by

$$W_{n,m} = \{t \in V_n : |\hat{t} \cap V_n| = m,\ n\in N,\ m = 0,1,\dots\}.$$

The set $W_{n,m}$ is an antichain, because it follows from $t_1 < t_2$, where $t_1, t_2 \in V_n$, that $|\hat{t}_1 \cap V_n| < |\hat{t}_2 \cap V_n|$. Thus $T = \bigcup_{n=1}^{\infty} \bigcup_{m=0}^{\infty} W_{n,m}$ and every $W_{n,m}$ is an antichain.

<u>Corollary 4.3</u>. Let $(T, \leq)$ be an Aronszajn tree. Then $C_p(X)$ is a Lindelöf Σ-space if and only if $(T,\leq)$ is special.

<u>Corollary 4.4</u>. Let $(T,\leq)$ be an S-bush (in particular, a Souslin tree). Then X is a Corson compact, for which $C_p(X)$ is not a Lindelöf Σ-space.

Notice that every S-bush contains a Souslin tree. The proof of this statement, in fact, could be easily extracted from [12]. Thus we have

<u>Theorem 4.5</u>. The existence of an S-bush is equivalent to the Souslin problem.

It is known that, by the theorem of J. Baumgartner, if Martin axiom plus $2^{\aleph_0} > \aleph_1$ (MA + $\neg$CH) is assumed, then every Aronszajn tree is special. A slight generalization of the W. Fleissner's proof [13, p. 18] allows us to establish the analogous assertion for A-bushes.

Theorem 4.6. (MA + $\neg$CH). Let $(T, \preceq)$ be an A-bush and $|T| < 2^{\aleph_0}$. Then T is a union of a countable family of antichains.

Nevertheless, there is an A-bush $(T, \preceq)$ with $|T| = 2^{\aleph_0}$ and which cannot be decomposed into a countable family of antichains. It is the matter of Example S.1.

An adequate compact constructed on an S-bush has some more properties.

Theorem 4.7. Let $(T, \preceq)$ be an S-bush. Let $\mathcal{A}$ be a family of its chains and $X = X_{\mathcal{A}}$. Then $d(X) = \aleph_1$, $c(X) = \aleph_0$.

Proof: X is a subspace of $\mathcal{D}^T$, then $d(X) \leq |T| = \aleph_1$. The converse inequality follows from the nonmetrizability of X. In order to prove the remaining part, according to [11, 3.3], it suffices to show that the partially ordered set $(P, \preceq)$ consisting of all finite elements of $\mathcal{A}$, partially ordered by $A \preceq B$ iff $B \subset A$, is ccc. Suppose, otherwise, that $\{A_\alpha\}_{\alpha<\omega_1}$ is an uncountable collection of pairwise incompatible elements of $(P, \preceq)$. Denote by $m_\alpha = \max\{t : t \in A_\alpha\}$. Since $(T, \preceq)$ contains no uncountable antichain, there are distinct $\alpha, \beta < \omega_1$ such that $m_\alpha < m_\beta$. Because $\hat{m}_\beta$ is a totally ordered subset of $(T, \preceq)$, it follows that $A_\alpha \cup A_\beta \subset \hat{m}_\beta$ and the elements A_α, A_β are compatible in $(P, \preceq)$. The contradiction proves the theorem.

As was shown by A.V. Arhangel'skiĭ (cf. [1]) the construction of such a Corson compact in the framework of ZFC is impossible.

__Theorem 4.8.__ Let X be an adequate compact. Then X is an Eberlein compact if and only if there is a partition $T = \bigcup_{i \in N} T_i$ such that $|\operatorname{supp} x \cap T_i| < \aleph_0$ for each $x \in X$ and $i \in N$.

Proof: (if). Denote by π_i the projection of X onto T_i. Then the diagonal product $\Delta \pi_i : X \to \prod_{i \in N} \pi_i(X)$ is the homeomorphic embedding of X into the countable product of strong Eberlein compacts. Hence, X is an Eberlein compact.

(only if). Clearly, X is the zero-dimensional compact. For the zero-dimensional Eberlein compact X the space $C_p(X, \mathfrak{D})$ has the type K_σ as it was observed by many authors ([5],[7]). T^* is closed in $C_p(X, \mathfrak{D})$, hence, T^* also has the type K_σ. Then $T^* = \bigcup_{i \in N} T_i \cup \{*\}$ and every $T_i \cup \{*\}$ is compact. This means that, if $A \subset T_i$ and $A \in \mathcal{O}\!\mathcal{l}$, then $|A| < \aleph_0$.

__Theorem 4.9.__ Let X be an adequate compact. Then X is a uniform Eberlein compact if and only if there is a partition $T = \bigcup_{i \in N} T_i$ and an integer-valued function $N(i)$ such that $|\operatorname{supp} x \cap T_i| < N(i)$ for each $x \in X$ and $i \in N$.

Proof: (if). The argument is the same as in the proof of Theorem 4.8 with the slight difference that $\pi_i(X)$ in this case is a uniform Eberlein compact.

(only if). We may assume that for each $t \in T$ the function $\chi_{\{t\}} \in X$. Then the set $S = \{\chi_{\{t\}}\}_{t \in T}$ is discrete and has a unique limit point $\mathbb{O}$ in X. According to [10, Lemma 3], there is a partition $T = \bigcup_{n \in N} \Gamma_n$ and neighborhoods U_t for each $\chi_{\{t\}}$,

$t \in T$ in X such that if $t_1, t_2, \dots, t_{n+1} \in \Gamma_n$ are distinct, then $\bigcap_{j=1}^{n+1} U_{t_j} = \emptyset$. Without loss of generality we can assume that each U_t is basic, i.e.

$U_t = \{y \in X : y(t) = 1,\ y|_{M_t} \equiv 0\}$, where $M_t \subset T$, $|M_t| < \aleph_0$.

Thus $\Gamma_n = \bigcup_{m=0}^{\infty} \Gamma_{n,m}$, where $\Gamma_{n,m} = \{t \in \Gamma_n : |M_t| = m\}$. Renumbering $\Gamma_{n,m}$, we obtain the partition $T = \bigcup_{i \in N} T_i$ and integer-valued functions $n(i)$ and $m(i)$ such that $U_t = \{y \in X : y(t) = 1,$ $y|_{M_t} \equiv 0\}$, where $M_t \subset T$, $|M_t| = m(i)$ for each $t \in T_i$, and $\bigcap_{j=1}^{n(i)} U_{t_j} = \emptyset$ for arbitrary distinct $t_1, t_2, \dots, t_{n(i)} \in T_i$. This partition is required. The function $N(i)$ may be chosen as follows: $N(i) = C^{m-1}_{m+n-2}$, where $m = (m(i)+1)^2 + 1$, $n = n(i)$. To prove this, suppose on the contrary that there exist $x \in X$ and $i \in N$ such that $|\operatorname{supp} x \cap T_i| \geq N(i)$. For every $A \subset \operatorname{supp} x \cap T_i$ with $|A| = n(i)$ there are distinct $t, s \in A$ such that $U_t \cap U_s = \emptyset$, otherwise, $\chi_A \in \bigcap_{t \in A} U_t$ in contradiction with $\bigcap_{t \in A} U_t = \emptyset$. Reformulate the situation to the language of the graph theory. We have a graph of $N(i)$ vertices. The vertices t and s are joined by an edge iff $U_t \cap U_s = \emptyset$. This graph has the property that for every $n(i)$-tuple of vertices there exists the pair of vertices which are joined by an edge. Then the Erdös-Szekeres's estimate for the Ramsey problem [15, p. 30] yields that there is a complete subgraph with m vertices. Since $m = (m(i)+1)^2 + 1$, it is easy to conclude that for some vertice t, $|M_t| > m(i)$ holds. This is a contradiction with our assumptions and the theorem is proved.

5. Examples. As has been noted, every adequate compact

is zero-dimensional. But any zero-dimensional Corson compact is not necessarily a compact constructed on some adequate family of sets. To see this it suffices to take a nonmetrizable first-countable zero-dimensional Corson compact, for instance, the Alexandroff double of the Cantor cube $\mathfrak{D}^{\aleph_0}$. If it were adequate, then in consequence of nonmetrizability, it would contain a one-point compactification of the uncountable discrete space in contradiction with the first axiom of countability.

Example 5.1. Let Q be the rationals. By σQ we denote the set of all bounded well ordered subsets of Q ordered as follows: $s < t$ iff S is a proper initial segment of t. σQ is clearly a tree without uncountable chains. Then by [14, Theorems 2.4, 3.3 (ii)] it follows that σQ is not special.

The second example described below is obtained by the "doubling" of the space constructed in [5].

Example 5.2. Let T be an arbitrary subset of the real line $\mathbb{R}$ with $|T| = \aleph_1$. It can be well ordered by the type ω_1. Define the partial ordering on T: $s < t$ iff s is less than t in both the reals and the ordinals order. Denote by $\mathcal{A}_1$ and $\mathcal{A}_2$ the families of all chains and antichains of $(T, \leq)$ respectively. It is well known [13, p. 8] that $\mathcal{A} = \mathcal{A}_1 \cup \mathcal{A}_2$ consists of at most countable sets, hence $X = X_{\mathcal{A}}$ is an adequate Corson compact. Let us observe that, according to the Ramsey theorem [13, p. 7], every infinite subset of T contains an infinite subset which belongs to the family $\mathcal{A}$. Show that T^* and consequently $C_p(X)$ fails to be a Lindelöf Σ-space. Suppose on the contrary that there is a family of compacts $\{F_n\}_{n\in N}$ from the Stone-Čech compactification $\beta(T^*)$, closed with respect to finite intersections, and such that for each point $x \in T^*$ the set

$B_x = \bigcap\{F_n : x \in F_n, n \in N\}$ is nonempty and contained in T^*. Every set B_x is compact, therefore it is finite, and since $|T| = \aleph_1$ it is easy to see that some B_x is different from any F_n. We can suppose that $B_X = \bigcap_{i \in N} F_{n_i}$, where $\{n_i\} \subset N$ and $F_{n_i} \supset F_{n_{i+1}}$, $F_{n_i} \neq B_X$ for each $i \in N$. Pick up points $x_i \in F_{n_i} \setminus F_{n_{i+1}}$. The set $\{x_i\}_{i \in N}$ is infinite, hence, it includes some infinite subset which belongs to $\mathcal{A}$. Without loss of generality we can assume that the set $\{x_i\}_{i \in N}$ has itself this property. Then, on the one hand, $\{x_i\}_{i \in N}$ is a discrete subset of T^* and, on the other hand, from

$$\emptyset \neq \bigcap_{k \in N} [\{x_i\},\ i \geq k\}]_{\beta(T^*)} \subset \bigcap_{i \in N} F_{n_i} = B_x \subset T^*$$

it follows that the set $\{x_i\}_{i \in N}$ has a limit point in T^*. This contradiction proves the assertion.

Example 5.3. Denote by Ω the set of all ordinal numbers less than ω_1 and put $T = \Omega \times \Omega$. Partial ordering on T is: $(\alpha_1, \beta_1) < (\alpha_2, \beta_2)$ iff $\alpha_1 < \alpha_2$, $\beta_1 > \beta_2$. Every chain of $(T, \leq)$ is finite. Indeed, if $\{t_i\}_{i \in N}$ is a chain, where $t_i = (\alpha_i, \beta_i)$, then we can assume that $\alpha_1 < \alpha_2 < \ldots$, hence $\beta_1 > \beta_2 > \ldots$ holds, which is impossible. $(T, \leq)$ has the following property: for any its partition at a countable family of subsets at least one of the subsets contains chains with any finite lengths. Let us prove this claim. If $T = \bigcup_{n \in N} A_n$ for each $\alpha \in \Omega$, $n \in N$, denote by $A_n^\alpha = \{\beta \in \Omega : (\alpha, \beta) \in A_n\}$. Then $\bigcup_{n \in N} A_n^\alpha = \Omega$ and $\omega_1 = \sup_{n \in N} \sup A_n^\alpha$. One easily sees that for each $\alpha \in \Omega$ there exists $n \in N$ such that $\sup A_n^\alpha = \omega_1$. Consequently, there exist $\Gamma \subset \Omega$, $|\Gamma| = \aleph_1$ and $n_0 \in N$ such that $\sup A_{n_0}^\alpha = \omega_1$, for every $\alpha \in \Gamma$. We claim that for every natural k the set A_{n_0} contains a chain with the length k. To prove this let us renumber naturally the first k elements of $\Gamma : \alpha_1 < \alpha_2 < \ldots\ \alpha_k$. Choose a point $\beta_k \in A_{n_o}^{k}$. Then sup $A_{n_o}^{\alpha_{k-1}} = \omega_1$ implies the existence of $\beta_{k-1} \in A_{n_o}^{\alpha_{k+1}}$ with $\beta_{k-1} > \beta_k$. Proceeding by induction we obtain the finite sequence $\beta_k < \beta_{k-1} < \ldots < \beta_1$, where $\beta_1 \in A_{n_o}^{\alpha_1}$. Clearly, $\{(\alpha_i, \beta_i)\}_{i=1}^{k}$ is the chain and is contained in A_{n_o}.

If X is an adequate compact constructed on the family of all chains of $(T, \leq)$ then, evidently, X is a strong Eberlein compact but it is not a uniform Eberlein compact by virtue of Theorem 4.9.

The same example shows that for an arbitrary partially ordered set Theorem 4.2 is not true.

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<u>Remarks</u>. Recently we have been informed that K. Alster and R. Pol proved that their example from [5] has the same properties as our Corson compact in Example 5.2.

Also, after this paper had been prepared for print, we discovered that D. Kurepa, in the paper Ensembles Ordonnes et Ramifies, Publ. Math. Univ. Belgrade 4(1935), introduced the notion of pseudotrees which coincide with one of our bushes. But our classes of bushes are investigated with other purposes.

R e f e r e n c e s

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