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ON SOME CLASSES OF COMPACT SPACES LYING IN Σ-PRODUCTS

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Abstract: In this paper results on the structure of perfect classes in the sense of A.V. Archangel'skii [1] are presented. We give topological characterizations of compact spaces X for which the space $C_p(X)$ of all continuous functions with the topology of pointwise convergence is a $\mathcal{K}$-analytic space or a Lindelöf Σ-space.

Key words: The space $C_p(X)$, $\mathcal{K}$-analytic spaces and Lindelöf Σ-spaces, perfect classes of compacts.

Classification: 54C35

I. A.V. Archangel'skiǐ [1] introduced the notion of the perfect class of topological spaces and proved the following remarkable theorem

($\ast$) Let $\mathcal{P}$ be a perfect class, X be a compact, $Y \subset C_p(X)$ be a subspace separating points of X and $Y \in \mathcal{P}$. Then $C_p(X) \in \mathcal{P}$.

Moreover, A.V. Archangel'skiǐ [1] has shown that the class $\mathcal{P}_1$ of all $\mathcal{K}$-analytic spaces and the class $\mathcal{P}_2$ of all Lindelöf Σ-spaces [1, 3, 6] are perfect. We need here some strengthening of the Archangel'skiǐ's definition of a perfect class (in [1] somewhat weaker than condition (A3) below is required). We do not know whether our definition is equivalent to the original one, but observe that all results of [1] hold in the new situation; the class $\mathcal{P}_1$ and the class $\mathcal{P}_2$ satisfy our modification and, actually, we are able to establish new facts on the structure of "perfect" classes in this regard. (See, for example, Remark 1.) Throughout this article we do not use the Archangel'skiĭ's definition, for this reason we venture to preserve the same name "perfect" for the next notion:

A class $\mathcal{P}$ of topological spaces is said to be perfect if it satisfies each of the following conditions.

(A1) $\mathcal{P}$ contains all compact spaces and the countable discrete space N,

(A2) if $X \in \mathcal{P}$ and Y is a continuous image of X or a closed subset of X then $Y \in \mathcal{P}$,

(A3) if $X_n \in \mathcal{P}$, $n \in N$, then $\prod_{n \in N} X_n \in \mathcal{P}$.

If $\mathcal{P}$ is a perfect class we denote by $\mathcal{C}(\mathcal{P})$ the class of all compact spaces X such that $X \in \mathcal{C}(\mathcal{P})$ if and only if $C_p(X) \in \mathcal{P}$. The main section III of this paper is devoted to a study of classes $\mathcal{C}_1 = \mathcal{C}(\mathcal{P}_1)$ and $\mathcal{C}_2 = \mathcal{C}(\mathcal{P}_2)$, where $\mathcal{P}_1$ and $\mathcal{P}_2$ are as above. We give the characterization of these classes which is similar to the Rosenthal's one of Eberlein compacts [4] and obtain some consequences of our characterization. Results in the section II are that the class $\mathcal{C}(\mathcal{P})$ is closed under some standard topological operations. We prove, in particular, that if a Corson compact X is a countable union of Eberlein compacts then $X \in \mathcal{C}_1$. The known example in this regard (M. Talagrand [5]) states that there exists a Corson compact with these properties which is not an Eberlein compact.

Our terminology and notations are standard and follow the previous author's and A.G. Leiderman's paper [6]. In particular, we denote by $\Sigma(T)$ the Σ-product of real lines having T as an index set.

II. Throughout this section $\mathcal{P}$ denotes some perfect class.

Theorem 1. The class $\mathcal{P}$ contains any countable union and any countable intersection of its elements.

Proof. a) Let X be a union of its subspaces X_n and $X_n \in \mathcal{P}$, $n \in N$. Then X is a continuous image of the discrete sum $Y = \bigoplus_{n \in N} X_n$ and Y is homeomorphic to the closed subspace of $(\prod_{n \in N} X_n) \times N$. Therefore, we obtain $X \in \mathcal{P}$ by (A1) - (A3).

b) Let X be a common subspace of $X_n \in \mathcal{P}$, $n \in N$, and $X = \bigcap_{n \in N} X_n$. Then X is naturally homeomorphic to the "diagonal" in $\prod_{n \in N} X_n$, hence X can be identified with a closed subspace of this product and $X \in \mathcal{P}$ by (A2) and (A3).

Remark 1. By (A1) and Theorem 1 $K_{\delta\sigma}$-spaces, hence by (A2) $\mathcal{K}$-analytic spaces belong to every perfect class $\mathcal{P}$. Therefore $\mathcal{P}_1$ is the smallest perfect class. Is it true for perfect classes in the sense of A.V. Archangel'skiĭ?

For a continuous mapping $\pi : X \to Y$, the induced continuous mapping $\pi_0 : C_p(Y) \to C_p(X)$ is defined by the formula $\pi_0(f) = f \circ \pi$.

Theorem 2. The class $\mathcal{E}(\mathcal{P})$ is closed under the following operations:

a) countable products,

b) finite unions,

c) countable intersections,

d) continuous images,

e) closed subspaces.

Proof. Let X be a product of $X_n \in \mathcal{E}(\mathcal{P})$ and $\pi_n : X \to X_n$ be a projection for each $n \in N$. Then the set $Y = \bigcup_{n \in N} \pi_n^{\circ}(C_p(X_n))$

lies in $C_p(X)$ and separates points of X. By ($*$) and Theorem 1 we have (a). Assertions d) and e) follow from ($*$), (A2) and the next general fact: π is an injection (a surjection) iff π^o is a surjection (an injection). To prove b) it is enough to remark that the map $f \mapsto (f|_{X_1}, f|_{X_2})$, where $X = X_1 \cup X_2$ and $f \in C_p(X)$, is a homeomorphism of $C_p(X)$ with a closed subspace of $C_p(X_1) \times C_p(X_2)$. Finally, the same reasoning as in the part b) of the proof of Theorem 1 proves the point c). Q.E.D.

It should be mentioned here that a generalization of the point b) of Theorem 2 for countable unions is false. Indeed, let $X = bN$ be a compactification of N whose remainder $bN \setminus N$ is homeomorphic to the one-point compactification of the uncountable discrete space. Then X is the union of $bN \setminus N$ and singletons $\{n\}$, $n \in N$; all of these sets are Eberlein compacts and therefore they are elements of $\mathcal{E}_1$. It is evident that X is separable and nonmetrizable, hence, X is not a Corson compact and by [7] $X \notin \mathcal{E}_1$. However, if we add to conditions on X to be a Corson compact then the generalization mentioned above is true. To prove this assertion we need some new facts concerning Corson compacts.

Let us fix a Corson compact X and some embedding of X into $\Sigma(T)$. One can consider the point $*$, $* \notin T$, and one can suppose that $x(*) = 0$ for every $x \in \Sigma(T)$. Moreover, we may assume that X separates points of $T \cup \{*\}$. We equip the set $T \cup \{*\}$ with the weakest topology for which any $x \in X$ is continuous. We denote the topological space determined in this manner by T_X. It is obvious that the next assertion holds.

<u>Lemma 1</u>. The mapping $\varkappa : T_X \to C_p(X)$ defined by $\varkappa(t)(x) = x(t)$, $t \in T_X$, $x \in X$, is a homeomorphism.

Lemma 2. There exists an open cover $\{T_X^\alpha; \alpha \in A\}$ of the set $T_X \setminus \{*\}$ consisting of at most countable discrete disjoint subsets of T_X such that

1) sets $[\varkappa(T_X^\alpha)]_{C_p(X)} \setminus \{0\}$ are disjoint in $C_p(X)$,

2) if $P \subset T_X$, $* \in P$ and $|P \cap T_X^\alpha| \leq 1$ for every $\alpha \in A$ then $\varkappa(P)$ is closed in $C_p(X)$.

Proof. Since X is a Corson compact by assumption, it follows from Gul´ko´s theorem [2; 3] that there is a linear injection $u: C_p(X) \longrightarrow \Sigma(S)$ for some set S. Observe that $u \circ \varkappa(*) = 0$ and put $Z = u \circ \varkappa(T_X)$. The topology of the space T_X is such that $T_X \setminus U$ is finite or countable for any neighbourhood U of the point $*$. It follows that sets $Z_s = \{z \in Z; z(s) \neq 0\}$, $s \in S$, are at most countable. Letting $\alpha_1(s) = \{s\}$, by induction we define

$$\alpha_{n+1}(s) = \bigcup\{\operatorname{supp} z; z \in Z_{s'},\ s' \in \alpha_n(s)\},\ n \geq 1,$$

$$\alpha(s) = \bigcup_{n \in N} \alpha_n(s),\ s \in S.$$

It is easy to see that the sets $\alpha(s)$ are countable and either disjoint or coincident. The last fact means that the sets $Z_{\alpha(s)} = \bigcup\{Z_{s'}; s' \in \alpha(s)\}$ are open and either disjoint or coincident, too. Finally, we put $T_X^{\alpha(s)} = (u \circ \varkappa)^{-1} Z_{\alpha(s)}$. It is easy to examine that the family of sets $T_X^{\alpha(s)}$ is desired.

Q.E.D.

Take now $\{t_n^\alpha; n \in N\}$ - some enumeration of T_X^α and put $T_X^n = \{t_n^\alpha; \alpha \in A\}$. The set T_X^n is discrete and $\varkappa(T_X^n) \cup \{0\}$ is closed in $C_p(X)$ by Lemma 2.

Theorem 3. Let X be a Corson compact. Then there exists a subspace $Y \subset C_p(X)$ which separates points of X and which is a union of closed (in $C_p(X)$) sets Y_n such that $Y_n \setminus \{0\}$ is discrete for each $n \in N$ and the sets $Y_n \setminus \{0\}$ are disjoint.

Proof. It suffices to put $Y = \varkappa(T_X)$ and $Y_n = \varkappa(T_X^n) \cup \cup \{0\}$, $n \in N$. Q.E.D.

Since $\varkappa(T_X)$ separates points of X, we infer from $(*)$ and Theorem 3:

Theorem 4. If X is a Corson compact then $C_p(X) \in \mathcal{P}$ iff $T_X \in \mathcal{P}$.

Definition 1. A set $X \subset \Sigma(T)$ is said to be order closed if it satisfies the next condition: $x \in X$ and $|y(t)| \leq |x(t)|$ for every $t \in T$ imply $y \in X$. The order envelope $oe(X)$ of $X \subset \Sigma(T)$ is the smallest order closed subset of $\Sigma(T)$ containing X.

Lemma 3. If $X \subset \Sigma(T)$ is compact then $oe(X)$ is compact, too.

Proof. One can consider spaces X and $oe(X)$ lying in $\mathbb{R}^T$ and verify that $oe(X)$ is closed in $\mathbb{R}^T$. Notice that X (and therefore $oe(X)$) in fact lies in some product of intervals $[-a_t, a_t]$, $t \in T$. This proves our assertion.

Lemma 4. If a compact $X \subset \Sigma(T)$ is order closed then $\varkappa(T_X)$ is closed in $C_p(X)$.

Proof. It easily follows from definitions of topologies in $C_p(X)$ and in T_X.

Theorem 5. Let X be a Corson compact and $X \in \mathscr{C}(\mathcal{P})$. Then for any embedding X into $\Sigma(T)$ the order envelope $oe(X)$ belongs to $\mathscr{C}(\mathcal{P})$, too.

Proof. oe X-topology on $T \cup \{*\}$ is a priori finer than

X-topology; but on $T_X^n \cup \{*\}$ these topologies coincide, because T_X^n is discrete and every oe X - neighbourhood of $*$ contains some X-neighbourhood. Thus the spaces T_X and $T_{oe(X)}$ are countable unions of the same subspace $T_X^n \cup \{*\}$, $n \in N$. The desired conclusion follows from Theorems 1 and 4.

Theorem 6. Let X be a Corson compact and X be a countable union of elements $\mathcal{E}(\mathcal{P})$. Then $X \in \mathcal{E}(\mathcal{P})$.

Proof. Let us fix an embedding $X \subset \Sigma(T)$ and put $X_0 = \sigma_1(T) = \{x \in \Sigma(T);\ |\operatorname{supp} x| \leq 1\}$. Then X_0 is homeomorphic to the one-point compactification of the discrete space T, therefore X_0 is an Eberlein compact and $X_0 \in \mathcal{E}(\mathcal{P})$ by Remark 1. If $X = \bigcup_{n \in N} X_n$ and $X_n \in \mathcal{E}(\mathcal{P})$, we can assume that $X_0 \subset X_1 \subset$ $\subset X_2 \subset \dots$ by Theorem 2(b). Every space X_n, $n = 0,1,\dots$, separates points of $T \cup \{*\}$ and these spaces give the increasing sequence of topologies on $T \cup \{*\}$ having X-topology as their upper bound. It follows that the diagonal product of mappings $\pi_n : T_X \longrightarrow T_{X_n}$, $n \in N$, is the homeomorphic mapping of T_X onto a closed subset of $\prod_{n \in N} T_{X_n}$. Using (A2), (A3) and Theorem 4, we complete the proof.

Corollary 1. If a Corson compact X is a countable union of Eberlein compacts then $X \in \mathcal{E}_1$.

III. We begin this section with some characterizations of $\mathcal{K}$-analytic spaces and Lindelöf Σ-spaces.

Definition 2. Let X be a topological space and γ be a family of open sets. The subset $Y \subset X$ is said to be γ-compact if some finite subfamily of γ covers Y.

Theorem 7. a) The space X is $\mathcal{K}$-analytic iff for every $n\in N$ and every n-tuple $(k_1,\dots,k_n)$ of integers there exists the closed subset $A_{k_1,\dots,k_n}\subset X$ such that $X=\bigcup_{k\in N} A_k$, $A_{k_1,\dots,k_n}=\bigcup A_{k_1,\dots,k_n,k}$ and the following is fulfilled: for any open cover γ of X and for any sequence $\{k_n\}_{n\in N}$ of integers the set $A_{k_1,\dots,k_n}$ is γ-compact for all sufficiently large $n\in N$.

b) The space X is a Lindelöf Σ-space iff there exists a countable family $\mathcal{A}$ of closed subsets of X such that for any open cover γ of X the subfamily $\{A\in\mathcal{A};\ A$ is γ-compact$\}$ covers X.

Proof. We prove only the part (b), because the part (a) can be verified by the same manner. If X is a Lindelöf Σ-space, there is a countable family $\mathcal{B}$ of closed subsets of βX (βX is the Stone-Čech compactification of X) such that $\mathcal{B}_x = \bigcap\{B\in\mathcal{B};x\in A\}\subset X$ for every $x\in X$. We can assume that $\mathcal{B}$ is closed under finite intersections. The family $\mathcal{A}=\{B\cap X;\ B\in\mathcal{B}\}$ satisfies all our requirements. Indeed, let γ be an open cover of X and $x\in X$. There are $G_1,\dots,G_n$ from γ, for which $\mathcal{B}_x\subset G_1\cup\dots\cup G_b$. It is evident that we have $\mathcal{B}_x\subset B\subset G_1\cup\dots\cup G_n$ for some $B\in\mathcal{B}$, therefore, $A=B\cap X$ is γ-compact and contains the point x. It completes the proof.

Definition 3. The family α is called weakly σ-point-finite if there is $\alpha_n\subset\alpha$ such that for every $x\in X$ we have $\alpha=\bigcup\{\alpha_n;n\in N_x\}$ where $N_x=\{n\in N;\ \alpha_n$ is finite at the point $x\}$. The family α is called T_0-separating points of X if for any two different points there is $\in\alpha$ containing only one of them.

The next two theorems are suggested by the well-known Rosenthal's theorem [4].

<u>Theorem 8</u>. Let X be a compact. Then the following are equivalent:

a) $X \in \mathcal{C}_2$,

b) X has a weakly σ-point-finite T_0-separating family of open F_σ subsets,

c) there is the embedding of X into $\Sigma(T)$ and $T_n \subset T$ for each $n \in N$ such that $T = \cup \{T_n; n \in N_x\}$ for every $x \in X$ where $N_x = \{n \in N; T_n \cap \operatorname{supp} x \text{ is finite}\}$.

<u>Proof</u>. a) $\Rightarrow$ b). Since X is a Corson compact we can assume that X is embedded in $\Sigma(T)$ in such a way that $0 \le x(t) \le 1$ for all $x \in X$ and $t \in T$. We can also assume (by Theorems 2(b) and 5) that $\{x \in \Sigma(T); |\operatorname{supp} x| \le 1\} \subset X$ and X is order closed. Let Q be a set of all rational numbers and define

$\alpha = \cup \{\alpha_r; r \in Q \cap (0,1)\}$;

$\alpha_r = \{U_{rt}; t \in T\}$;

$U_{rt} = \{x \in X; x(t) > r\}$.

It is clear that each U_{rt} is an open F_σ set and α is T_0-separating. Moreover, it is easy to check that α is a weakly σ-point-finite family as any α_r is the same. Let $M_r: X \to X$ be a mapping defined as follows: $M_r(x)(t) = x(t)$ if $x(t) \ge r$, otherwise $M_r(x)(t) = 0$. It is evident that $M_r(X)$ is a compact and $M_r(X) \subset X$. Thus $M_r(X)$ is a closed subset of X, hence, $M_r(X) \in \mathcal{C}_2$ and $T_{M_r(X)} \in \mathcal{P}_2$ by Theorem 4. By Theorem 6(b) there is the countable family $\mathcal{A}_2 = \{A_{rn}; n \in N\}$ of closed subsets of $T_{M_r(X)}$ such that

(1) $\qquad \cup \{A \in \mathcal{A}_r; A \text{ is } \tau\text{-compact}\} = T_{M_r(X)}$

for any open cover γ of $T_{M_r(X)}$. We put $\alpha_{rn} = \{U_{rt} \in \alpha_r;$ $t \in A_{rn}\}$ and show that the sequence α_{rn} satisfies all conditions of Definition 3. To this end let us fix $x \in X$ and an open cover γ of $T_{M_r(X)}$ as follows: γ consists of the set $T_{M_r(X)} \setminus \operatorname{supp} M_r(x)$ and singletons $\{t\}$, $t \in \operatorname{supp} M_r(x)$. Then the family of all γ-compact sets A_{nr} covers $T_{M_r(X)}$. However, it is easy to check that A_{rn} is γ-compact iff $A_{rn} \cap \operatorname{supp} M_r(x)$ is finite iff α_{rn} is finite at the point x. Therefore, the condition (1) is equivalent to the equality

$$\alpha_r = \bigcup \{\alpha_{rn};\ \alpha_{rn} \text{ is finite at the point } x\},$$

as required.

(b) $\Longrightarrow$ (c). Let $f_u : X \to [0,1]$ be a continuous function such that $f_u^{-1}(0) = X \setminus U$ for each $U \in \alpha$. Define $\pi : X \longrightarrow \Sigma(\alpha)$ by $\pi(x)(U) = f_u(x)$. Then π is a homeomorphism. It is easy to see that π, $T = \gamma$ and $T_n = \alpha_n$, $n \in N$, satisfy (c).

(c) $\Longrightarrow$ (a). If X satisfies (c) then oe(X) satisfies it, too. We will assume that X is order closed itself. It remains to verify that $T_X \in \mathcal{P}_2$. To this end define sets $A_\sigma = \bigcap\{T_k;$ $k \in \sigma\}$ where σ runs over the set L of all finite subsets of N. Let γ be any open cover of T_X. By the definition of the topology in T_X we can assume that γ consists of the set $T_X \setminus \bigcup_{k=1}^{m} \operatorname{supp} x_k$ and singletons $\{t\}$, $t \in \bigcup_{k=1}^{m} \operatorname{supp} x_k$ for some $x_1, \dots, x_m$ in X. For $t \in T_X$ by (c) we can find $n_k \in N$ such that $T_{n_k} \cap \operatorname{supp} x_k$ is finite for $k = 1, \dots, m$. Denoting $\sigma = \{n_1, \dots$ $\dots, n_m\}$ we have $t \in A_\sigma$ and A_σ is γ-compact. Thus, we get a cover of T_X consisting of γ-compact sets. By Theorem 7(b) it suffices for the proof.

The main stages of the following theorem are similar to Theorem 8 and for this reason its proof is omitted.

Theorem 9. Let X be a compact. Then the following are equivalent:

a) $X \in \mathcal{E}_1$,

b) X has a T_0-separating family γ of open F_σ subsets and subfamilies $\gamma_{k_1 \dots k_n} \subset \gamma$ for any n-tuple $(k_1, \dots, k_n)$ of integers such that

1) $\gamma = \bigcup_{k \in N} \gamma_k$, $\gamma_{k_1 \dots k_n} = \bigcup_{k \in N} \gamma_{k_1 \dots k_n k}$,

2) for every $x \in X$ and any sequence $\{k_n\}_{n \in N}$ of integers there is n_0 such that $\gamma_{k_1 \dots k_n}$ is finite at the point x for $n \geq n_0$.

c) There is an embedding X into $\Sigma(T)$ in such a way that for some subsets $T_{k_1 \dots k_n} \subset T$, the following conditions are fulfilled:

1) $T = \bigcup_{k \in N} T_k$, $T_{k_1 \dots k_n} = \bigcup_{k \in N} T_{k_1 \dots k_n k}$,

2) If $\{k_n\}$ is a sequence of integers and $x \in X$ then the set $T_{k_1 \dots k_n} \cap \operatorname{supp} x$ is finite for all sufficiently large n.

Remark 2. It should be noted that part (c) of Theorems 7 and 8 give us a new information even for Eberlein compacts.

Corollary 2. Let $X \in \mathcal{E}_2$. Then there exists a countable family $\mathcal{A}$ of closed subsets of X such that $\bigcap \{A \in \mathcal{A}; x \in A\}$ is an Eberlein compact for every $x \in X$.

Proof. We can assume that X satisfies (c) of Theorem 8, i.e. there is T and $T_n \subset T$ for $n \in N$ such that $X \subset \Sigma(T)$, $T = \bigcup \{T_n; n \in N_x\}$ for every $x \in X$ where N_x is the set of all $n \in N$

for which $T_n \cap \operatorname{supp} x$ is finite. We can assume that X separates points of $T \cup \{*\}$. Let R_n be a retraction in $\Sigma(T)$ defined by $R_n(x)(t) = x(t)$ if $t \in T_n$, otherwise $R_n(x)(t) = 0$. Denote $\sigma_k(T) = \{x \in \Sigma(T), |\operatorname{supp} x| \le k\}$ and $\mathcal{A} = \{R_n^{-1}\sigma_k(T); n \in N, k \in N\}$. It is evident that all elements of $\mathcal{A}$ are compact. If $x \in X$ and $n \in N_x$, then R_n maps $\mathcal{A}_x = \cap\{A \in \mathcal{A}; x \in A\}$ into a σ-product of real lines, therefore, $R_n(\mathcal{A}_x)$ is an Eberlein compact. Since $T = \cup\{T_n; n \in N_x\}$, the family of mappings R_n, $n \in N_x$, separates points of $\mathcal{A}$. It follows that the diagonal product of R_n is a homeomorphism of $\mathcal{A}_x$ into the countable product of Eberlein compacts $R_n(\mathcal{A}_x)$, $n \in N_x$, hence, $\mathcal{A}_x$ is an Eberlein compact too, as it is required.

Corollary 2 allows us to state next questions:

(1) Is the condition of Corollary 2 sufficient for $X \in \mathcal{C}_2$?

(2) (S.P. Gul'ko). Given $X \in \mathcal{C}_1$, do there exist Eberlein compacts X_{nk} and Corson compacts X_n such that $X_n = \bigcup_{k \in N} X_{nk}$ and X is a continuous image of a closed subset of $\prod_{n \in N} X_n$?

R e f e r e n c e s

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