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APPROXIMATIONS OF σ-CLASSES AND π-CLASSES

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Abstract: This paper is a contribution to the development of the alternative set theory. We define π-classes (and σ-classes similarly) relatively w.r.t. a codable class $\mathfrak{M}$ (so called $\pi^{\mathfrak{M}}$-classes and $\sigma^{\mathfrak{M}}$-classes). If Q is a $\pi^{\mathfrak{M}}$-class then there is a relation $R \in \mathfrak{M}$ with $\mathrm{dom}(R) \in N$ such that $Q = \bigcap \{R''\{n\}; n \in FN\}$ (so called π-string of Q). This description of $\pi^{\mathfrak{M}}$-classes enables us, in the case if $\mathfrak{M}$ is rich enough, to approximate a $\sigma^{\mathfrak{M}}$-class Q in the following sense: if Q has a property of a certain type then there is a π-string $R \in \mathfrak{M}$ of Q such that the classes $R''\{\alpha\}$ have an analogous one. An exact form of this proposition can be found in the theorems 2.0.1, 2.0.2.

Key words: π-class, σ-class, standard system, down-hereditary formula, up-hereditary formula, alternative set theory.

Classification: 02K10, 02K99

Introduction. If Q is a π-semiset then Q is a "uniform π"-class in the following sense: there is a set-relation r with $\mathrm{dom}(r) \in N$ such that $Q = \bigcap \{r''\{n\}; n \in FN\}$. (We say that r is a π-string of Q.) This uniformity is very useful for a work with π-semisets. There is a natural question whether every π-class Q is a "uniform π"-class in the sense that there is a set-theoretically definable π-string of Q. We prove that there is a π-class which is no "uniform π"-class.

Moreover, we shall define a notion of $\mathcal{T}$-class relatively w.r.t. a codable class $\mathfrak{M}$ (so called $\mathcal{T}^{\mathfrak{M}}$-class) so that each $\mathcal{T}^{\mathfrak{M}}$-class will have a $\mathcal{T}$-string which is an element of $\mathfrak{M}$. Specifying $\mathfrak{M}$ as a rich enough class (the so called standard system) we can treat $\mathcal{T}^{\mathfrak{M}}$-classes with advantage. Note that every $\mathcal{T}$-class is a $\mathcal{T}^{\mathfrak{M}}$-class where $\mathfrak{M}$ is any revealment of the codable class Sd_V. (See 0.0.1, 1.0.4.) Our description of $\mathcal{T}^{\mathfrak{M}}$-classes enables us to approximate each $\mathcal{T}^{\mathfrak{M}}$-class Q in the following sense: if Q satisfies a property of a certain type then there is a $\mathcal{T}$-string of Q such that $R \in \mathfrak{M}$ and the classes $R''\{\alpha\}$ satisfies an analogous one. (See 2.0.1, 2.0.2.)

§ 0. Preliminaries

0.0.0. The class of all natural numbers (finite natural numbers resp.) is denoted by N (FN resp.). We use $\alpha, \beta, \gamma, \delta, \xi, \vartheta$ (m,n,i,j,k resp.) as variables ranging over natural (finite natural resp.) numbers. RN is the class of rational numbers. We shall use lower-case letters to denote sets.

The operation of composition of relations is denoted by $\circ$. The symbol Id denotes the identity mapping. Writing $H: :X \longrightarrow Y$ we mean that H is a function with dom(H) = X and $rng(H) \subseteq Y$.

0.0.1. Sd_V denotes the codable class of all set-theoretically definable classes. Writing Sd_V^* we mean that Sd_V^* is a revealment of Sd_V. (See [S-V2].) The codable class of all classes set-theoretically definable without parameters is denoted by Sd_o.

0.1.0. Let $\mathfrak{M}$ be a codable class. Writing $FL_{\mathfrak{M}}$ we mean a language FL_K such that there is a relation S so that $\langle S,K\rangle$ is a coding pair which codes the class $\mathfrak{M}$. It is obvious, how is defined the satisfaction of the formulas of the language $FL_{\mathfrak{M}}$ (cf. [S1]). Let φ be a formula of the language $FL_{\mathfrak{M}}$. Writing $\varphi(x_0,\dots,x_m)$ we mean that the formula φ has no free variables distinct from $x_0,\dots,x_m$. Let $T_0,\dots,T_k$ be terms of the language $FL_{\mathfrak{M}}$. We let

$$\varphi\left(\frac{T_0}{X_{i_0}},\dots,\frac{T_k}{X_{i_k}}\right)$$

designate the formula obtained from φ by replacing all free occurences of $X_{i_0},\dots,X_{i_k}$ by $T_0,\dots,T_k$ resp. We shall omit the subscripts $X_{i_0},\dots,X_{i_k}$ when they are immaterial or clear from the context. If there is no danger of confusion we shall not make a distinction between a class $X\in\mathfrak{M}$ and the constant denoting this class.

Let φ be a formula of the language $FL_{\mathfrak{M}}$. The symbol $\varphi^{(\mathfrak{M})}$ denotes the formula resulting from φ by restriction of all quantifiers binding class-variables to elements of $\mathfrak{M}$. Suppose that φ is a sentence of the language $FL_{\mathfrak{M}}$. The sentence "φ holds in the sense of $\mathfrak{M}$" denotes that $\varphi^{(\mathfrak{M})}$ holds.

0.2.0. Recall that a class X is a <u>σ-class</u> (a <u>π-class</u> resp.) iff X is the union (the intersection resp.) of a countable sequence of set-theoretically definable classes.

§ 1. <u>$\sigma^{\mathfrak{M}}$-classes and $\pi^{\mathfrak{M}}$-classes and their basic properties</u>

1.0.0. A codable class $\mathfrak{M}$ is called a <u>standard system</u> iff the following holds:

(1) $V \subseteq \mathfrak{M}$

(2) Let $\varphi(x)$ be a normal formula of the language $FL_{\mathfrak{M}}$. Then $\{x; \varphi(x)\} \in \mathfrak{M}$.

(3) Let $X \in \mathfrak{M}$ be a class such that $0 \neq X \subseteq N$. Then there exists the least element of X.

Evidently, the codable class Sd_V of all set-theoretically definable classes is a standard system. Moreover, $Sd_V \subseteq \mathfrak{M}$ holds for every standard system $\mathfrak{M}$.

Throughout this paper let $\mathfrak{M}$ denote a standard system.

1.0.1. Proposition. (1) No proper semiset is an element of $\mathfrak{M}$.

(2) Each axiom of GB_{fin} holds in the sense of $\mathfrak{M}$. (GB_{fin} denotes the theory obtained from GB by substituting the axiom of infinity by its negation.)

(3) Each class of $\mathfrak{M}$ is fully revealed.

Proof. (1) Let $X \neq 0$ be a semiset of $\mathfrak{M}$. We put $A = \{f;\ f$ is a one-one mapping & $\mathrm{dom}(f) \in N$ $\mathrm{rng}(f) \subseteq X\}$. Clearly, $A \in \mathfrak{M}$ holds. We define $B = \{\alpha; (\exists f \in A)(\mathrm{dom}(f) = \alpha)\}$. We have $B \in \mathfrak{M}$ and B is a semiset. Let γ be the greatest element of B. Thus, there is a one-one mapping f such that $\mathrm{dom}(f) = \gamma$ and $\mathrm{rng}(f) \subseteq X$. Suppose that $\mathrm{rng}(f) \subseteq X$. Let $x \in X -$ $- \mathrm{rng}(f)$. Thus, the function $f \cup \{\langle x, \gamma \rangle\}$ is an element of A, which is a contradiction. Consequently, $X = \mathrm{dom}(f)$ and X is a set.

(2) It follows from (1) that only the following proposition must be proved: If $F \in \mathfrak{M}$ is a function and u is a set then $F''u$ is a set. Suppose that $F \in \mathfrak{M}$ is a function and u is a set. We put $B = \{v \subseteq u; (\exists t)(F''v \subseteq t)\}$. Clearly, $B \in \mathfrak{M}$ and consequently, B is a subset of P(u). Let v be a $\subseteq$ -maximal

element of B. We deduce from the maximality of v that $v = u$. Thus, there is a set t such that $F"u \subseteq t$. Moreover, $F"u \in \mathfrak{M}$ and, consequently, F"u is a set.

(3) Let X be a class of $\mathfrak{M}$. Let $S \subseteq X$ be a countable class. Then there is a function f such that $f \restriction FN$ is a one-one mapping of FN on S. Put $A = \{\alpha \in dom(f); f(\alpha) \in X\}$. We have $A \in \mathfrak{M}$ and, consequently, A is a set. Clearly, $S \subseteq f"A \subseteq X$. We deduce from this that X is a revealed class. Thus each class of $\mathfrak{M}$ is revealed and the proposition (3) follows immediately from this.

1.0.2. <u>A string</u> is a relation R such that $dom(R) \in N$. A string R is called a <u>σ (π resp.)-string</u> iff $R"\{\alpha\} \subseteq R"\{\alpha+1\}$ ($R"\{\alpha+1\} \subseteq R"\{\alpha\}$ resp.) holds for each $\alpha+1 \in dom(R)$. A <u>σ (π resp.)-string of a class</u> X is a σ (π resp.)-string R such that $\bigcup\{R"\{n\}; n \in FN\} = X$ ($\bigcap\{R"\{n\}; n \in FN\} = X$ resp.).

Let R be a string. We shall write $R(\alpha)$ instead of $R"\{\alpha\}$.

A class X is called <u>$\sigma^{\mathfrak{M}}$-class</u> (<u>$\pi^{\mathfrak{M}}$-class</u> resp.) iff there exists a string $R \in \mathfrak{M}$ such that $X = \bigcup\{R(n); n \in FN\}$ ($X = \bigcap\{R(n); n \in FN\}$ resp.).

The following is obvious:

(a) X is a $\sigma^{\mathfrak{M}}$-class ($\pi^{\mathfrak{M}}$-class resp.) iff there exists a σ-string (π-string resp.) R of X and $R \in \mathfrak{M}$.

(b) X is a $\sigma^{\mathfrak{M}}$-class iff $V - X$ is a $\pi^{\mathfrak{M}}$-class.

(c) Let X be a semiset. X is a σ-class (π-class resp.) iff X is a $\sigma^{\mathfrak{M}}$-class ($\pi^{\mathfrak{M}}$-class resp.). (For the notion of the σ- (π-resp.) class see 0.2.0.)

1.0.3. <u>Proposition</u>. (1) Each $\pi^{\mathfrak{M}}$-class is revealed.

(2) A $\pi^{\mathfrak{M}}$-class X is a π-class iff X is a real class.

(3) A $\sigma^{\mathfrak{M}}$-class X is a σ-class iff X is a real class.

Proof. (1) follows from the fact that each $\pi^{\mathfrak{M}}$-class is the intersection of a countable sequence of revealed classes. (2) The part "only if" follows from the fact that each π-class is real. The part "if" follows from (1) and from the following proposition: every real revealed class is a π-class. (3) follows immediately from (2).

Remark. For the notion of a real class and the facts used in the previous proof see [Č-V 1].

1.0.4. We shall write σ^{o} (π^{o} resp.) instead of the symbol σ^{Sd_V} (π^{Sd_V} resp.). Thus, a class X is a σ^{o} (π^{o} resp.)-class iff X is a $\sigma^{\mathfrak{M}}$ ($\pi^{\mathfrak{M}}$ resp.)-class for each standard system $\mathfrak{M}$. Let Sd_V^* be a revealment of Sd_V (see [S-V 2]). We have $Sd_V \subseteq Sd_V^*$ and, for each sequence $\{X_n; n \in FN\} \subseteq Sd_V^*$, there is a relation $R \in Sd_V$ with $(\forall n)(R''\{n\} = X_n)$ (see [S-V 2]). We deduce from this that each σ (π resp.)-class is a $\sigma^{Sd_V^*}$ ($\pi^{Sd_V^*}$ resp.)-class.

We shall prove that there is a σ-class which is not a σ^{o}-class. Let us recall that the following proposition holds: there is no relation $R \in Sd_V$ such that $(\forall Y \in Sd_o)(\exists y)(Y = R''\{y\})$. (See [S-V 2].) At first, we shall strengthen it.

1.0.5. Proposition. (1) There is no relation R such that (a) R is a σ^{o}-class, (b) $(\forall Y \in Sd_o)(\exists y)(Y = R''\{y\})$.

(2) There is no relation R such that

(a) R is a π^{o}-class, (b) $(\forall Y \in Sd_o)(\exists y)(Y = R''\{y\})$.

Proof. (1) Suppose that there is a relation R such that (a),(b) hold. Let $\Phi(x,y,z)$ be a normal formula of the language FL_V such that $\langle x,y\rangle \in R \equiv (\exists n)\Phi(x,y,n)$. Let $\{Y_n\}_{n \in FN}$ be a numbering of Sd_o. Let us choose, for each $n \in FN$, a set y_n such that $Y_n = R''\{y_n\}$. We have $x \in Y_n \equiv (\exists n)\Phi(x,y_n,n)$. We shall prove that there is a $m \in FN$ such that $x \in Y_n \equiv (\exists \alpha \leq m)$ $\Phi(x,y_n,\alpha)$. Suppose that $(\forall m)(\exists x)(x \in Y_n \,\&\, (\forall \alpha \leq m)$ $\neg \Phi(x,y_n,\alpha))$. Let H be a function on FN such that, for each $m \in FN$, $H(m) \in Y_n \,\&\, (\forall \alpha \leq m) \neg \Phi(H(m),y_n,\alpha)$ holds. Let $h \supseteq H$ be a function which is a set. Thus, $(\forall m)(h(m) \in Y_n \,\&$ $\&\, (\forall \alpha \leq m) \neg \Phi(h(m),y_n,\alpha))$ holds. We deduce from this that there is a $\gamma \in N - FN$, $\gamma \in dom(h)$ and $h(\gamma) \in Y_n \,\&\, (\forall \alpha \leq \gamma)$ $\neg \Phi(h(\gamma),y_n,\alpha)$. Consequently, $(\forall m) \neg \Phi(h(\gamma),y_n,m)$ holds. But this is a contradiction, because $h(\gamma) \in Y_n$. Thus, $(\exists m)(\forall x)(x \in Y_n \longrightarrow (\exists \alpha \leq m)\Phi(x,y_n,\alpha))$ holds and, finally, there is a $m \in FN$ such that $x \in Y_n \equiv (\exists \alpha \leq m)\Phi(x,y_n,\ )$.

Let f be a function such that $dom(f) \supseteq \{y_n\}_n$ and $x \in Y_n \equiv$ $\equiv (\exists \alpha \leq f(y_n))\Phi(x,y_n,\alpha)$ holds for each $n \in FN$. We define the relation S as follows: $\langle x,y \rangle \in S \equiv (\exists \alpha \leq f(y))\Phi(x,y,\alpha)$. Obviously, $S \in Sd_V$. We deduce from the construction of S that $(\forall Y \in Sd_0)(\exists y)(Y = S''\{y\})$ holds, which is a contradiction. (2) follows from (1) immediately.

1.0.6. <u>Proposition</u>. Let $\{Y_n\}_{n \in FN}$ be a numbering of Sd_0 and let $A = \bigcup\{Y_n \times \{n\}; n \in FN\}$. Then A is a σ-class which is not a σ^0-class.

Proof. Clearly, A is a σ-class. We have $(\forall Y \in Sd_0)$ $(\exists y)(Y = A''\{y\})$. We deduce from the previous proposition that A is not a σ^0-class.

1.0.7. The equivalence $\doteq$ on V is defined as follows: $x \doteq y$ iff for each set-formula $\varphi(z)$ in FL we have $\varphi(x) \equiv$ $\equiv \varphi(y)$. $\doteq$ is an indiscernibility equivalence and each $Y \in Sd_0$ is a clopen figure in the equivalence $\doteq$. (See [V].)

<u>Proposition</u>. The equivalence $\doteq$ is not a π^0-class.

Proof. Suppose that $\doteq$ is a π^0-class. Let $\varphi(x,y,z)$

be a set-formula of the language FL_V satisfying: $x \stackrel{\circ}{=} y \equiv$ $\equiv (\forall n)\varphi(x,y,n)$. We put $\langle x,y\rangle \in S \equiv (\exists z \in y)(x \stackrel{\circ}{=} z)$. We have $\langle x,y\rangle \in S \equiv (\exists z \in y)(\forall n)\varphi(x,z,n) \equiv (\forall n)(\exists z \in y)(\forall \alpha \leq n)$ $\varphi(x,z,\alpha)$ and, consequently, S is a π^{0}-class. The $\stackrel{\circ}{=}$ is an indiscernibility equivalence. We deduce from this that for each closed figure Y exists a set y such that $Y = S''\{y\}$. Each class $Y \in Sd_0$ is a closed figure in $\stackrel{\circ}{=}$. Thus, $(\forall Y \in Sd_0)(\exists y)$ $(Y = S''\{y\})$ holds, which is a contradiction. (See 1.0.5.)

§ 2. Approximations of $\sigma^{\mathfrak{M}}$-classes and $\pi^{\mathfrak{M}}$-classes

2.0.0. A formula φ of the language $FL_{\mathfrak{M}}$ is down-hereditary (up-hereditary resp.) in a variable Z iff the general closure of the following formula holds:

$$(\forall X,Y)((X \subseteq Y \,\&\, \varphi(\tfrac{Y}{Z})) \longrightarrow \varphi(\tfrac{X}{Z}))$$
$$((\forall X,Y)((Y \subseteq X \,\&\, \varphi(\tfrac{Y}{Z})) \longrightarrow \varphi(\tfrac{X}{Z}))\ \text{resp.}$$

Let $\varphi(X_1,\dots,X_k)$ be a formula of the language FL and let A be a constant denoting a class of $\mathfrak{M}$. Writing $\varphi^{Ⓐ}(X_1,\dots,X_k)$ we mean the formula $\varphi(A-X_1,\dots,A-X_k)$. Obviously, for each i, $1 \leq i \leq k$, the formula φ is down-hereditary (up-hereditary resp.) in the variable X_i iff $\varphi^{Ⓐ}$ is up-hereditary (down-hereditary resp.) in the variable X_i.

Proposition. Let $\varphi(Z)$ be a normal formula of the language $FL_{\mathfrak{M}}$ down (up resp.)-herediatry in the variable Z. Let $R \in \mathfrak{M}$ be a σ-string (π-string resp.) of Q. Suppose that $\varphi(Q)$ holds. Then there is a $n \in FN$ such that $\varphi(R(n))$ holds.

Proof. 1. Let R be a σ-string of Q and let $dom(Q) = \xi$. We have $(\forall \alpha \in \xi - FN)\varphi(R(\alpha))$. Put $B = \{\alpha \in \xi;\ \varphi(R(\alpha))\}$. We deduce that $B \in \mathfrak{M}$ and $\xi - FN \subseteq B$. Thus $B \cap FN \neq 0$ and, consequently, there is a $n \in B \cap FN$ such that $\varphi(R(n))$ holds.

2. Let R be a π-string of Q. Let $\langle x, \alpha \rangle \in S \equiv \langle x, \alpha \rangle \notin R$. Then $S \in \mathfrak{M}$ and S is a σ-string of V-Q. We deduce from $\varphi^{Ⓥ}(V-Q)$ and from 1. that there is a $n \in FN$ such that $\varphi^{Ⓥ}(V-R(n))$ and, consequently, $\varphi(R(n))$ holds.

We say that a formula φ of the language $FL_{\mathfrak{M}}$ is $\langle X,Y \rangle$-<u>hereditary</u> iff φ is down-hereditary in the variable X and up-hereditary in the variable Y. Evidently, φ is $\langle X,Y \rangle$-hereditary iff $\varphi^{Ⓐ}$ is $\langle Y,X \rangle$-hereditary.

2.0.1. <u>Theorem</u>. Let $\varphi(X,Y)$ be a normal formula of the language $FL_{\mathfrak{M}}$ which is $\langle X,Y \rangle$-hereditary. Let Q be a $\sigma^{\mathfrak{M}}$-class and suppose $\varphi(Q,Q)$.

Then there is a σ-string R of Q, $R \in \mathfrak{M}$, such that the formula $\varphi(R(\alpha),((\alpha+1))$ holds for each $\alpha+1 \in dom(R)$.

Proof. Let S be a σ-string of Q, $S \in \mathfrak{M}$ and let $dom(S) = \xi$. We deduce from the previous proposition that

$(\forall m)(\exists n)(n > m \mathbin{\&} \varphi(S(m),S(n)))$. $\quad (*)$

Thus, there is a $\vartheta \in N-FN$ with $(\forall \alpha \in \vartheta)(\exists \beta \in \xi)(\beta > \alpha \mathbin{\&} \varphi(S(\alpha),S(\beta)))$. We put for each $\alpha \in \vartheta$: $G(\alpha) = \min\{\beta \in \xi;\ \beta > \alpha \mathbin{\&} \varphi(S(\alpha),S(\beta))\}$.

The G is a function, $G: \vartheta \to \xi$, and $G \in \mathfrak{M}$. Thus, G is a set. We deduce from $(*)$ that $G''FN \subseteq FN$. Let H be a function defined recursively on FN as follows: $H(0) = 0$, $H(n+1) = G(H(n))$. Let $h \supseteq H$ be a function. We have $(\forall n)(h(n+1) = G(h(n)) \mathbin{\&} h(n) \in \vartheta)$. Thus there is a $\alpha \in N-FN$ such that $(\forall \alpha \in \delta)(h(\alpha+1) = G(h(\alpha)) \mathbin{\&} h(\alpha) \in \vartheta$. We obtain from this that, for each $\alpha \in \delta$,

$$\varphi(S(h(\alpha)),S(h(\alpha+1))) \qquad (**)$$

holds. Put $\langle x, \alpha \rangle \in R \equiv \alpha \in \delta \mathbin{\&} \langle x, h(\alpha) \rangle \in S$. R is a σ-

string and $R \in \mathfrak{M}$. We have $G"FN \subseteq FN$ and, consequently, $h"FN \subseteq FN$ holds. We deduce from this that R is a σ-string of Q. Finally, we deduce $\varphi(R(\alpha), R(\alpha+1))$, for each $\alpha+1 \in$ $\in \operatorname{dom}(R)$, from $(**)$.

2.0.2. Theorem. Let $\varphi(X,Y)$ be a normal formula of the language $FL_{\mathfrak{M}}$ which is $\langle X,Y\rangle$-hereditary. Let Q be a $\pi^{\mathfrak{M}}$-class such that $\varphi(Q,Q)$ holds.

Then there is a π-string R of Q, $R \in \mathfrak{M}$, such that the formula $\varphi(R(\alpha+1), R(\alpha))$ holds for each $\alpha+1 \in \operatorname{dom}(R)$.

This follows from the previous theorem considering the class V-Q and the formula $\varphi^{\textcircled{V}}(X,Y)$.

2.1.0. Let $k \in FN$. Let, for each $i \leq k$, R_i be a $a(i)+1$-ary relation, $R_i \in \mathfrak{M}$ and $a(i) \in FN$. We denote by $[\![R_i]\!]_{i \leq k}(X,Y)$ the formula

$$R_0"X^{a(0)} \subseteq Y \,\&\, \ldots \,\&\, R_k"X^{a(k)} \subseteq Y.$$

Obviously, $[\![R_i]\!]_{i \leq k}(X,Y)$ is a normal formula of the language $FL_{\mathfrak{M}}$, which is $\langle X,Y\rangle$-hereditary.

Proposition. Let k, R_i, $i \leq k$, be as above and let $B \subseteq Q \subseteq$ $\subseteq A$ be classes such that $B \in \mathfrak{M}$, $A \in \mathfrak{M}$ and $[\![R_i]\!]_{i \leq k}(Q,Q)$ holds.

(1) Let Q be a $\sigma^{\mathfrak{M}}$-class. Then there exists a σ-string S of Q such that $S \in \mathfrak{M}$, $S(0) = B$, $S(\operatorname{dom}(S)-1) = A$ and $[\![R_i]\!]_{i \leq k}(S(\alpha), S(\alpha+1))$ holds for each $\alpha+1 \in \operatorname{dom}(S)$.

(2) Let Q be a $\pi^{\mathfrak{M}}$-class. Then there exists a π-string S of Q such that $S \in \mathfrak{M}$, $S(0) = A$, $S(\operatorname{dom}(S)-1) = B$ and $[\![R_i]\!]_{i \leq k}(S(\alpha+1), S(\alpha))$ holds for each $\alpha+1 \in \operatorname{dom}(S)$.

Proof. (1) Let $\varphi(X,Y)$ designate the formula $[\![R_i]\!]_{i \leq k}(X,Y) \,\&\, B \subseteq X \,\&\, Y \subseteq A$. We deduce from 2.0.1 that there exist a number $\xi \in N$ and a σ-string R of Q such that $R \in \mathfrak{M}$,

$\xi = \mathrm{dom}(R)$ and $\varphi(R(\alpha), R(\alpha+1))$ holds for each $\alpha+1 \in \xi$. Let S be a relation with the following properties: $\mathrm{dom}(S) = \xi$, $S''\{0\} = B$, $S''\{\xi - 1\} = A$ and, for each $1 \leq \alpha < \xi - 1$, $S''\{\alpha\} = R''\{\alpha + 1\}$. The σ-string in question is the S. (2) follows similarly as (1).

R e f e r e n c e s

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