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GENERALIZED PERIODIC SOLUTIONS OF NONLINEAR TELEGRAPH EQUATIONS

(Preliminary Communication)

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Abstract: We state the existence theorems for generalized solutions of nonlinear telegraph equations. This improves the earlier results from this field.

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Consider the generalized periodic solutions of nonlinear telegraph equation of the form

(1) $\beta u_t + u_{tt} - u_{xx} - \mu u^+ + \mu u^- + \psi(u) = h(t,x)$

(where $\beta \neq 0$, μ, ν are real parameters, ψ is a continuous and bounded real function and h is square Lebesgue integrable over I^2 with $I = [0,2\pi]$) the study of which was initiated in [2].

A generalized periodic solution of (1) (shortly GPS) is a real function $u \in L_2(I^2)$ (with the usual inner product $(\cdot,\cdot)$), such that, for all real C^2-functions v on I^2 which are 2π-periodic in both variables, one has

(2) $(u, -\beta v_t + v_{tt} - v_{xx}) = (\mu u^+ - \nu u^- - \psi(u) + h, v)$.

Our main results are summarized in the following three theorems.

Theorem 1. Let $\mu = \nu = q^2$ where q is nonnegative integer. Let ψ be a continuous bounded and odd function. Suppose that there exists $a > 0$ such that

(3) $\psi(\xi) > 0$ for $\xi \geqq a$ if $q = 0$,

(4) $\lim_{\xi \to \infty} \xi^2 \min_{\tau \in [a,\xi]} \psi(\tau) = \infty$ if $q = 1,2,\ldots$.

Then (1) has at least one GPS provided

(5) $$\int_0^{2\pi}\int_0^{2\pi} h(t,x)dx\, dt = 0 \text{ if } q = 0,$$

(6) $$\int_0^{2\pi}\int_0^{2\pi} h(t,x)\sin(qx+\varphi)dx\, dt = 0$$

for arbitrary $\varphi \in (-\infty,\infty)$ if $q = 1,2,\ldots$.

Before formulating the next result we introduce the following definition (see [1]). The bounded continuous non-trivial and odd function ψ is said to be expansive if for each p with

$$0 \leqq p < \sup_{\xi \in \mathbb{R}^1} \psi(\xi),$$

there exist sequences $0 < a_k < b_k$, with

$$\lim_{k \to \infty} \frac{b_k}{a_k} = \infty,$$

such that

$$\lim_{k \to \infty} \min_{\xi \in [a_k, b_k]} \psi(\xi) > p.$$

(Examples of expansive functions are given in [1].)

Theorem 2. Let $\mu = \nu = q^2$ for some $q = 0,1,2,\ldots$. Let ψ be an expansive function. Then (1) has at least one GPS provided

(7) $$\left|\int_0^{2\pi}\int_0^{2\pi} h(t,x)dx\, dt\right| < (2\pi)^2 \sup_{\xi \in \mathbb{R}^1} \psi(\xi) \text{ if } q = 0,$$

(8) $$\sup_{\varphi \in \mathbb{R}^1} \left| \int_0^{2\pi}\int_0^{2\pi} h(t,x)\sin(qx+\varphi)\,dx\,dt \right| <$$
$$< 8\pi \sup_{\xi \in \mathbb{R}^1} \psi(\xi) \text{ if } q = 1,2,\dots .$$

The reader is invited to sketch a picture of the set $\mathfrak{M}$ in the following theorem.

Theorem 3. Put

$$\mathfrak{M} = \{(\mu,\nu) \in \mathbb{R}^2;\ \mu<0,\ \nu<0\} \cup \bigcup_{k=0}^{\infty} \{(\mu,\nu) \in \mathbb{R}^2;$$
$$\mu^{1/2} > \tfrac{k}{2},\ \omega_k(\mu^{1/2}) < \nu^{1/2} < \omega_{k+1}(\mu^{1/2})\},$$

where

$$\omega_k(\tau) = \frac{k\tau}{2\tau - k}, \quad \tau \in (\tfrac{k}{2}, \infty).$$

If $(\mu,\nu) \in \mathfrak{M}$ then (1) has at least one GPS for any $h \in L_2(I^2)$.

The results were obtained during the time of Czechoslovak Conference on Differential Equations and Their Applications "EQUADIFF 4", August 22-26, 1977, Prague, Czechoslovakia. The proofs will appear later in Nonlinear Analysis.

R e f e r e n c e s

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