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A NOTE ON SEPARATION BY LINEAR MAPPINGS

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Abstract: Recently K.H. Elster [1] and R. Nehse [2] have introduced a concept of separation of two convex sets by linear mappings. The purpose of this note is to illustrate how these results can be extended to finite families of convex sets.

Key-words: Separation of convex sets, ordered linear space, linear mapping.

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Theorem. *Let* L *be a real linear space and* $(P,\leq)$ *a real ordered linear space. If there is* $y\in P$ *such that* $y>0$ *then for each finite family* $\{A_i : i\in I\}$ *of convex subsets of* L *such that each* A_i *has nonempty intrinsic core* A_i^o *and* $\bigcap_{i\in I} A_i^o = \emptyset$ *there exists a family* $\{y_i : i\in I\}$ *of points in* P *and a family* $\{F_i : i\in I\}$ *of linear mappings of* L *to* P *with the following properties :*

(1) $A_i \subset \{x\in L \mid F_i(x)\leq y_i\}$ *for every* $i\in I$,

(2) $\sum_{i\in I} F_i = 0$ *and* $\sum_{i\in I} y_i \leq 0$,

(3) *there is* $i\in I$ *such that* $F_i \neq 0$.

Proof. By the separation theorem of [3] there is a family $\{f_i : i\in I\}$ of linear functionals on L and a family $\{\lambda_i : i\in I\}$ of real numbers such that

$$A_i \subset \{ x \in L \mid f_i(x) \leq \lambda_i \} \text{ for every } i \in I,$$

$$\sum_{i \in I} f_i = 0 \text{ and } \sum_{i \in I} \lambda_i \leq 0,$$

$$f_i \neq 0 \text{ for some } i \in I.$$

Defining

$$F_i(x) = f_i(x) y, \quad y_i = \lambda_i y,$$

where y is a fixed element of P with $y > 0$, one obtains the required results by applying the rules (for $z \in P$ and real numbers λ, μ):

$$\lambda \neq 0 \text{ and } z \neq 0 \Longrightarrow \lambda z \neq 0,$$

$$\lambda \leq 0 \text{ and } z > 0 \Longrightarrow \lambda z \not\geq 0,$$

$$\lambda \leq \mu \text{ and } z > 0 \Longrightarrow \lambda z \not\geq \mu z.$$

References

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