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C O R R E C T I O N TO MY PAPER "EXISTENCE THEOREMS FOR OPERATOR EQUATIONS AND NONLINEAR ELLIPTIC BOUNDARY-VALUE PROBLEMS", Comment.Math. Univ.Carolinae 14(1973),27-46

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Professor P. Hess has remarked that the first part of relation (3.5) (Theorem 2)

$$(*)\quad B(0,u_0,u_0)=\sum_{|\beta|\le m-1}[B_\beta(\cdot,\xi_{m-1}(u_0)),D^\beta u_0]$$

is not proved.

We define $D(B)$ as in the paper but only for all $v\in W:=V\cap W_{m^*,n}$ and ask for a solution of (3.1) for all $v\in W$.

We assume the following additional condition on $F(x,\xi_{m-1},\xi'_{m-1})$ (see Assumption 5): To each $u\in V$ and $K>0$ there exists a function $F_{u,K}\in L^1(\Omega)$ such that

$$|F(x,\xi_{m-1}(u),\xi_{m-1}(v))|\le F_{u,K}(x)$$

a.e. on Ω for all $v\in V$ with $|v(x)|\le K$. Further let $m=1$.

Under this additional assumptions relation $(*)$ can be proved. Indeed, for each $K>0$ we define

$$\Omega_K := \{x \in \Omega : |u_0(x)| \le K\}$$

and set

$$u_0^K(x) := \begin{cases} u_0(x), & x \in \Omega_K \\ 0, & x \in \Omega \setminus \Omega_K \end{cases} .$$

Then $u_0^K \in V$ and $u_0^K \longrightarrow u_0$ in V as $K \longrightarrow \infty$. Following the pattern of the proof of Hess (J.Math.Anal.Appl. 43 (1973),241-249, p.248) for each K there exists a sequence $\{u_\nu\} \subset C_0^\infty(\Omega)$ satisfying $u_\nu \longrightarrow u_0^K$ in V, $|u_\nu(x)| \le K$ and $u_\nu(x) \longrightarrow u_0^K(x)$ a.e. on Ω. Hence $B(0, u_0, u_\nu) \longrightarrow B(0, u_0, u_0^K)$. From $B(0, u_0, u_\nu) = [B_0(\cdot, u_0), u_\nu]$ it follows by Assumption 5 (b) and the additional condition on F that the Theorem of Lebesgue is applicable. Hence

$$B(0, u_0, u_0^K) = [B_0(\cdot, u_0), u_0^K] .$$

By definition of u_0^K and Assumption 5 (a) the Theorem of Lebesgue again can be applied and (*) with $m = 1$ follows as $K \longrightarrow \infty$.

Relation (*) is unsolved for the case $m > 1$. The same remark holds for Theorem 2 in "Nonlinear eigenvalue problems, Comment.Math.Univ.Carolinae 14(1973),113-126.

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