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Label: Article

Jahr: 1974

PURL: https://resolver.sub.uni-goettingen.de/purl?316342866_0015|log36

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SOME PROPERTIES OF A GENERALIZED HEAT POTENTIAL

(Preliminary communication)

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Abstract: A generalized heat potential and its continuous extension from an open set with non-smooth boundary to its closure is studied.

Key words: generalized heat potential, boundary behaviour

AMS: 31B10

Ref.Ž.: 7.972.26

For $x = [x_1, \dots, x_{m+1}] \in \mathbb{R}^{m+1}$, $m \geq 3$ we shall write $x = [\hat{x}, x_{m+1}] = [x, t]$ where $x \in \mathbb{R}^m$, $t \in \mathbb{R}^1$. Similarly for the differential operator $\nabla = [\partial_1, \dots, \partial_{m+1}]$ we put $\hat{\nabla} = [\partial_1, \dots, \partial_m]$. Let G be the function defined on \mathbb{R}^{m+1} by

$$G(x) = x_{m+1}^{-\frac{m}{2}} \cdot \exp(-\|\hat{x}\| / 4x_{m+1}) \text{ for } x_{m+1} > 0,$$

$$G(x) = 0 \text{ for } x_{m+1} \leq 0.$$

Suppose D is an open set in \mathbb{R}^{m+1} with the boundary B for which $B_\tau = B \cap \{[x, t] \in \mathbb{R}^{m+1}, t \leq \tau\}$ is compact for any $\tau \in \mathbb{R}^1$.

\mathcal{C} will denote the collection of all bounded continuous functions on B and \mathcal{D} will be the space of all infinitely

differentiable functions φ with compact support
 $\text{spt } \varphi \in \mathbb{R}^{m+1}$.

For any $x \in \mathbb{R}^{m+1}$ and $\varphi \in \mathcal{D}(x) = \{\varphi \in \mathcal{D}; x \notin \text{spt } \varphi\}$
we define

$$T\varphi(x) = - \int_D (\hat{\nabla}_w G(x-w) \cdot \hat{\nabla} \varphi(w) + G(x-w) \partial_{m+1} \varphi(w)) dw.$$

The integral on the right-hand side is finite for any
 $\varphi \in \mathcal{D}(x)$. As $T\varphi(x)$ depends on values of φ in a
neighborhood of boundary B only we can define $T\varphi(x)$
even for any $\varphi \in \mathcal{D}$ by means of

$$T\varphi(x) \stackrel{\text{def}}{=} T\tilde{\varphi}(x)$$

where $\tilde{\varphi} \in \mathcal{D}(x)$ and $\varphi(x) = \tilde{\varphi}(x)$ in a neighborhood
of B . $T\varphi(x)$ may be considered as a distribution over
 \mathcal{D} and it is closely connected with classical heat poten-
tials of single and double layer.

Three following questions are solved:

(1) When there is a measure ν_x such that

$$T\varphi(x) = \int \varphi d\nu_x = \langle \varphi, \nu_x \rangle$$

for every $\varphi \in \mathcal{D}(x)$?

Replacing φ by f we can define $Tf(x) = \langle f, \nu_x \rangle$
for any $f \in \mathcal{C}$ provided ν_x from (1) exists.

(2) When $Tf(x)$ is a well-defined function of the va-
riable x on D for any $f \in \mathcal{C}$?

(3) When this function Tf defined on D can be conti-
nuously extended from D to $D \cup B$ for any $f \in \mathcal{C}$?

The case $m = 1$ was investigated for special D by M. Dont in [1] and similar questions were solved by J. Král in [2],[3] and by the author in [4].

Recall that for a measurable set $M \subset \mathbb{R}^{m+1}$ its perimeter $P(M)$ is defined by

$$P(M) = \sup_{\omega} \int_M \operatorname{div} \omega(w) dw$$

where $\omega = [\omega_1, \dots, \omega_{m+1}]$ ranges over system of all functions with components $\omega_i \in \mathcal{D}$, $i = 1, 2, \dots, m+1$ satisfying

$$\sum_{i=1}^{m+1} \omega_i^2(w) \leq 1, \quad w \in \mathbb{R}^{m+1}$$

Put $\Gamma = \{x \in \mathbb{R}^m; \|x\| = 1\}$, $Z = (0, \infty) \times \Gamma$. We define for any $z = [x, t]$ and $(\varphi, \gamma, \theta) \in (0, \infty) \times (0, \infty) \times \Gamma$

$$S_z(\varphi, \gamma, \theta) = \left[\hat{x} + \varphi\theta, x_{m+1} - \frac{\varphi^2}{4\gamma} \right].$$

Given $(\gamma, \theta) \in Z$ let $S(\gamma, \theta)$ be the parabola described by $S_z(\cdot, \gamma, \theta)$ on $(0, \infty)$. A point $l \in S = S(\gamma, \theta)$ is termed a hit of the parabola S on D provided each neighborhood of l meets both $S \cap D$ and $S - D$ in a set of positive H_1 -measure where H_k is the k -dimensional Hausdorff measure. The number of all hits of $S(\gamma, \theta)$ on D will be denoted by $n(z, \gamma, \theta)$. We put for any $z \in \mathbb{R}^{m+1}$

$$v(z) = \int_Z e^{-\gamma} \gamma^{\frac{m}{2}-1} n(z, \gamma, \theta) dH_m((\gamma, \theta)).$$

The function v which is called the parabolic variation of D is a lower semicontinuous function on \mathbb{R}^{m+1} .

The answers to questions (1) - (3) can be formulated now in a form of necessary and sufficient conditions corresponding to (1) - (3) as follows:

- (1) $v(x) < \infty$,
- (2) $P(D_\tau) < \infty$ for all $\tau \in \mathbb{R}^1$ where
 $D_\tau = D \cap \{[x, t] \in \mathbb{R}^{m+1}; t < \tau\}$,
- (3) $\sup \{v(\xi); \xi \in B_\tau\} < \infty$ for all $\tau \in \mathbb{R}^1$.

Complete proofs of the formulated results and some further details are contained in a paper submitted for the publication in Czechoslovak Mathematical Journal.

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(Oblatum 22.4.1974)