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ON FULL EMBEDDINGS OF CATEGORIES OF ALGEBRAS INTO CATEGORIES OF FUNCTORS WITH THIN DOMAIN

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(Preliminary communication)

Following [1], a category K is said to be binding if every category of universal algebras can be fully embedded into it.

<u>Definition</u>. A small category $\mathcal{k}$ is said to be <u>rich</u> if the category $\mathcal{S}^{\mathcal{k}}$ (of all functors from $\mathcal{k}$ into the category of sets) is binding.

In [3],[4], various questions concerning rich monoids are studied. The aim of the present note is to present two theorems concerning rich thin x) categories.

<u>Theorem 1</u>. Let $\mathcal{k}$ be a finite thin category. Let M, a non-trivial monoid without a non-trivial (i.e., non-identical) idempotent be given. Then the following properties of $\mathcal{k}$ are equivalent:

(1) $\mathcal{k}$ is rich.

(2) $\mathcal{S}^{\mathcal{k}}$ contains $\aleph_1$ non-isomorphic rigid objects xx).

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x) We recall that a category is said to be thin if there is at most one morphism with given domain and range.

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xx) An object a is called rigid if there is no nonidentical $\alpha : a \to a$.

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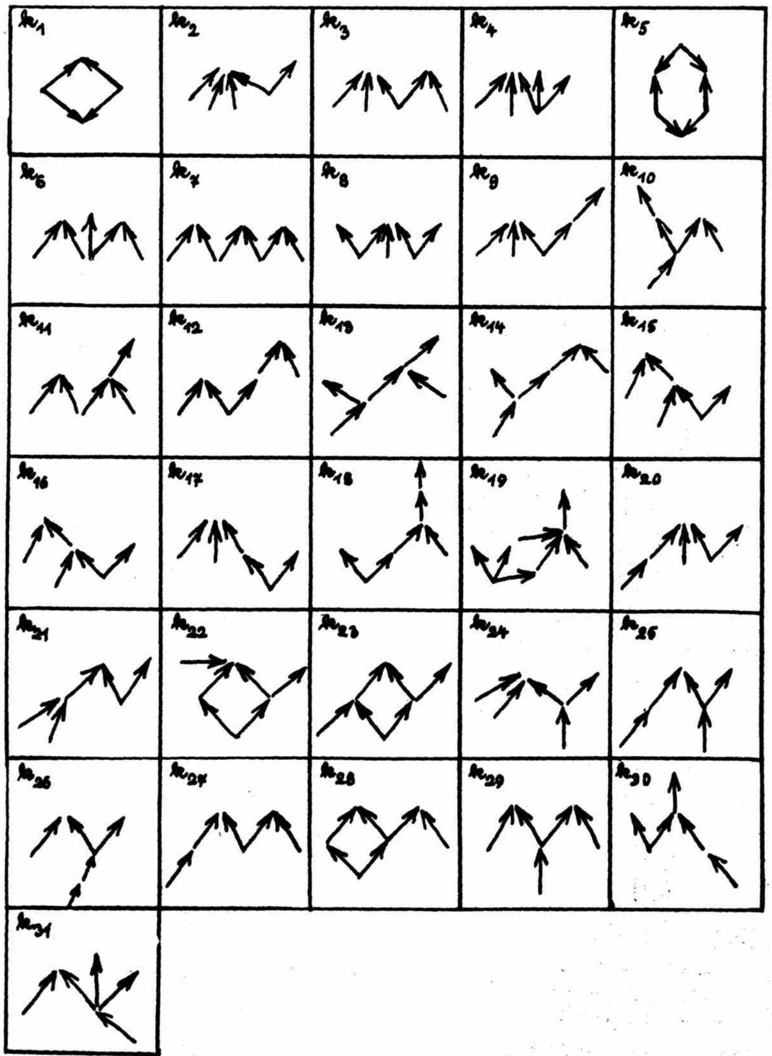
(3) M can be fully embedded into $\mathcal{S}^{\mathfrak{k}}$.

(4) Some one from the following categories $\mathfrak{k}_1, \dots, \mathfrak{k}_{31}$ is a full subcategory of $\mathfrak{k}$ (the identities and the composed morphisms are not indicated:



Definition. We say that a category $\mathfrak{K}$ is a category with trivial composition if either α or β is an identity whenever the composition $\alpha \circ \beta$ of morphisms α, β is defined.

Theorem 2. Let $\mathfrak{K}$ be a small thin category with trivial composition. Then the assertions (1) - (4) from the previous theorem are also equivalent. (Now, of course, $\mathfrak{K}_9$ - $\mathfrak{K}_{31}$ in (4) are superfluous.)

References

[1] Z. HEDRLÍN, A. PULTR: On full embeddings of categories of algebras, Ill.J.of Math.10(1966), 392-406.

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[3] J. SICHLER: Concerning minimal primitive classes of algebras containing any category of algebras as a full subcategory, Comment.Math. Univ.Carolinae 9(1968),627-635.

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