By Proposition 9, Chap. III, § 3 of [2] we have $\mu^+(1) - \mu^+(f^+) = \mu^+(1 - f^+) > 0$ and therefore $\mu(f) \leqq \mu^+(f^+) + \mu^-(f^-) < \mu^+(1) + \mu^-(1) = \|\mu\|$. This is a contradiction.

(ii) If $f(a) > 1$ then $\mu^-(1 - f^-) = \mu^-(1) - \mu^-(f^-) > 0$ and we obtain a contradiction similarly as in the preceding case.

Proof of Theorem. (i) First we shall prove that if $\mu \in M(\langle 0, 1\rangle)$ and $S(\mu^+) = S(\mu^-) = \langle 0, 1\rangle$, then ν and (λ_n) from the statement of Theorem do not exist. Suppose on the contrary that μ, ν, (λ_n) with the properties mentioned above are given. For a sufficiently large n we obtain easily that

$$S((\mu + \lambda_n\nu)^+) \neq \emptyset \quad \text{and} \quad S((\mu + \lambda_n\nu)^-) \neq \emptyset .$$

By Proposition, $S((\mu + \lambda_n\nu)^+) \cap S((\mu + \lambda_n\nu)^-) = \emptyset$ and therefore there exists an open interval $I \subset \langle 0, 1\rangle$ such that $I \cap S((\mu + \lambda_n\nu)^+) = \emptyset$ and $I \cap S((\mu + \lambda_n\nu)^-) = \emptyset$. Let $f \in C(\langle 0, 1\rangle)$ be a function with its support in I. If $k \neq n$, then

$$(1) \qquad (\mu + \lambda_k\nu)(f) = (\mu + \lambda_n\nu)(f) + (\lambda_k - \lambda_n)\,\nu(f) = (\lambda_k - \lambda_n)\,\nu(f) .$$

Since $(\mu + \lambda_n\nu)(f) = \mu(f) + \lambda_n\,\nu(f) = 0$ we have $\nu(f) = -\mu(f)/\lambda_n$. Thus we obtain from (1) $(\mu + \lambda_k\nu)(f) = \lambda_n^{-1}(\lambda_n - \lambda_k)\,\mu(f)$. Therefore we have $S((\mu + \lambda_k\nu)^+) \cap I = S((\mu + \lambda_k\nu)^-) \cap I = I$ and this is a contradiction with Proposition.

(ii) We shall prove that the set

$$A = M(\langle 0, 1\rangle) \setminus \{\mu \in M(\langle 0, 1\rangle);\ S(\mu^+) = S(\mu^-) = \langle 0, 1\rangle\}$$

is a set of the first category in $M(\langle 0, 1\rangle)$. In fact,

$$A = \bigcup\{A^+_{rs} \cup A^-_{rs};\ r < s \text{ and } r, s \text{ are rational}\} ,$$

where A^+_{rs} and A^-_{rs} are the sets of all measures $\mu \in M(\langle 0, 1\rangle)$ for which $S(\mu^+) \cap (r, s) = \emptyset$ and $S(\mu^-) \cap (r, s) = \emptyset$, respectively. The sets A^+_{rs}, A^-_{rs} are obviously closed nowhere dense subsets of $M(\langle 0, 1\rangle)$. Theorem is proved.

References

[1] *E. Bishop, R. R. Phelps:* A proof that every Banach space is subreflexive, Bull. Amer. Math Soc. *67* (1961), 97—98.

[2] *N. Bourbaki:* Éléments de Mathématique, Livre VI, Intégration, Paris.

Author's address: 186 00 Praha 8 - Karlín, Sokolovská 83 (Matematicko-fyzikální fakulta UK).