$$\supset (\mathsf{R} \setminus \bigcup_{i=1}^{n} D_i) \cap \{(\mathsf{R} \setminus I_{n+1}) \cup D_4^{n+1}\} \cap J =$$
$$= (D_4^{n+1} \cap J) \cup \{(\mathsf{R} \setminus \bigcup_{i=1}^{n} D_i) \cap (\mathsf{R} \setminus I_{n+1}) \cap J\},$$

is a set of the second Baire category. Therefore the sets $D_1, D_2, \ldots, D_n, D_{n+1}$ satisfy properties 1), 2), 3), 4), and 5) and the proof of the lemma is complete.

We now proceed to the proof of the existence of a function $g$ with the properties mentioned in our introduction.

**Theorem.** *There exists a Lebesgue measurable function $g$, $g : \mathsf{R} \to \mathsf{R}$ such that $\{x \in I;\ g(x) \in J\}$ is a set of the second Baire category in $\mathsf{R}$ for each non-empty open interval $I$ and each set $J$ which is of the second Baire category in $\mathsf{R}$.*

Proof. There exists a sequence of sets $\{D_n\}_{n=1}^{\infty}$ satisfying the five properties mentioned in the last lemma. By Corollary in the introduction we can express $D_n$ as $D_n = \bigcup_{j<c} D_{n,j}$, where each set $D_{n,j}$ is of the second Baire category in $\mathsf{R}$ and such that the sets $\{D_{n,j}\}_{j<c}$ are pairwise disjoint. Here $\mathfrak{c}$ denotes the cardinal of the continuum. Let $\{a_j\}_{j<c}$ be an enumeration (i.e. a well-ordering) of $\mathsf{R}$. Define $g(x) = a_j$ for each $x \in \bigcup_{n=1}^{\infty} D_{n,j}$ and $g(x) = 0$ for each $x \notin \bigcup_{n=1}^{\infty} D_n$. Then $g$ satisfies the requirements of our theorem, in fact $\{x \in I;\ g(x) = y\}$ is a set of the second Baire category in $\mathsf{R}$ for each real number $y$ and each non-empty open interval $I$. The function $g$ is clearly measurable as each set $D_n$ has Lebesgue measure zero.

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