Proof. There exist $r > 0$ and a positive integer N_1 such that

$$S(p, r) = (y; |y - p| < r) \subset G_2 \quad \text{and} \quad \alpha_n S(p, r) \subset G_1 \quad \text{if} \quad n \geqq N_1 .$$

Notice that if $n \geqq N_1$, we have

$$[\alpha_n S(p, r) \setminus P_1] \cap \alpha_n[S(p, r) \setminus P_2] \subset A \cap \alpha_n B \quad \text{or}$$
$$\alpha_n S(p, r) \setminus P_1 \setminus \alpha_n P_2 \subset A \cap \alpha_n B \quad \text{for} \quad n = N_1, N_1 + 1, \ldots .$$

Furthermore, there exists a positive integer N_2 such that

$$\alpha_n S(p, r) \supset S(q, (rq/2p)) \quad \text{if} \quad n \geqq N_2 \quad (\text{since } \lim_{n \to \infty} \alpha_n = \alpha = q/p) .$$

Therefore if $n \geqq N = \max(N_1, N_2)$ we have
(∗) $A \cap \alpha_n B \supset S(q, (rq/2p)) \setminus P_1 \setminus \alpha_n P_2$.
P_1 and $\alpha_n P_2$ are sets of the first Baire category.
Inclusion (∗) implies that

$$A \cap \alpha_n B \supset S(q, (rq/2p)) \setminus [P_1 \cup \bigcup_{k=N}^{\infty} \alpha_k P_2] \quad \text{if} \quad n \geqq N ,$$

or we obtain that

$$X \supset S(q, (rq/2p)) \setminus [P_1 \cup \bigcup_{k=N}^{\infty} \alpha_k P_2] ,$$

or X is a set of the second Baire category, which completes the proof.

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