

Werk

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cycle is a loop) and by $\lambda(C)$ we denote the maximal length of a directed path in this component which contains no edge of the cycle (it may be 0, if C consists only of a cycle).

Now we shall prove a theorem which gives a characterization of the finite graphs $G(S, a)$.

Theorem. *Let G be a finite directed graph in which each vertex has the outdegree 1. The graph G is isomorphic to the graph $G(S, a)$ for a semigroup S and its element a if and only if it contains a connected component C with the property that for each connected component D of G the number $\kappa(D)$ divides $\kappa(C)$ and $\lambda(D) \leq \lambda(C) + 1$.*

Proof. Suppose that G is isomorphic to $G(S, a)$ for some S and a . Let a have a period h and a pre-period k ; this means that the elements a, a^2, \dots, a^{h+k-1} are pairwise distinct and $a^{h+k} = a^k$. Hence in G there exists a cycle of the length h and a directed path of the length k whose terminal vertex belongs to this cycle; the initial vertex of this path corresponds to the element a . Now let x be an arbitrary vertex of G (i.e. an element of S); let D be the connected component of G containing x . Let p be the length of the directed path outgoing from x , incoming into a vertex of a cycle and containing no edge of this cycle; evidently $p \leq \lambda(D)$. Let $q = \kappa(D)$. Then the elements $x, xa, xa^2, \dots, xa^{p+q-1}$ are pairwise distinct and $xa^{p+q} = xa^p$. If q does not divide h , then k and $h+k$ are not congruent modulo q and thus $xa^k \neq xa^{h+k}$, which is a contradiction with the assumption $a^{h+k} = a^k$. Hence q must divide h and h is $\kappa(C)$, where C is the connected component of G containing a . Now suppose $p \geq k + 2$. Then xa^{k+1} is distinct from xa^l for each $l \neq k + 1$. But, as $a^{h+k} = a^k$, we must have $xa^{k+1} = xa^{h+k+1}$, which is a contradiction. Hence $p \leq k + 1 \leq \lambda(C) + 1$. Thus the necessity of the condition is proved.

Now suppose that the condition is fulfilled. In C take a directed path containing no edge of a cycle and having the length $\lambda(C)$; its initial vertex will be a . Take all sources of G and if G contains connected components distinct from C which are cycles, choose one vertex in each of them. The set thus obtained will be denoted by B . The vertex a and the vertices of B will be considered elements of a semigroup S . Each remaining vertex will be denoted as a power of a or a product of an element of B with a power of a in the way corresponding to the definition of $G(S, a)$. Further, we introduce the equality $xb = b$ for each $x \in S$ and each $b \in B$. Thus we have defined a semigroup S such that G is isomorphic to $G(S, a)$.

Reference

- [1] Teh, H. H. - Shee, S. C.: Algebraic Theory of Graphs. Singapore 1976.

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