Časopis pro pěstování matematiky, roč. 106 (1981), Praha

GRAPHS OF SEMIGROUPS

BOHDAN ZELINKA, Liberec

(Received October 5, 1979)

Analogously to graphs of groups (see e.g. [1]) we shall introduce graphs of semigroups.

Let S be a semigroup, let A be its subset. The graph $G(S, A)$ is a directed graph whose vertices are elements of S and in which there is a directed edge from a vertex u into a vertex v if and only if $v = ua$, where $a \in A$.

Here we shall characterize finite graphs $G(S, A)$, where A is a one-element set. Thus we shall have $A = \{a\}$ and instead of $G(S, \{a\})$ we shall write simply $G(S, a)$. We shall admit loops and consider them as cycles of the length 1.

Every graph $G(S, a)$ has the property that the outdegree of each of its vertices is 1. The structure of such graphs is well-known. If such a graph is finite, then each of its connected components contains exactly one cycle (by a cycle we mean a directed circuit).

After deleting all edges of this cycle a forest is obtained. Each tree of this forest has the property that for each of its vertices there is a directed path going from this vertex to a vertex of the cycle (Fig. 1). If C is a connected component of such a graph, then by $\varkappa(C)$ we denote the length of the cycle contained in C (it may be 1, if this

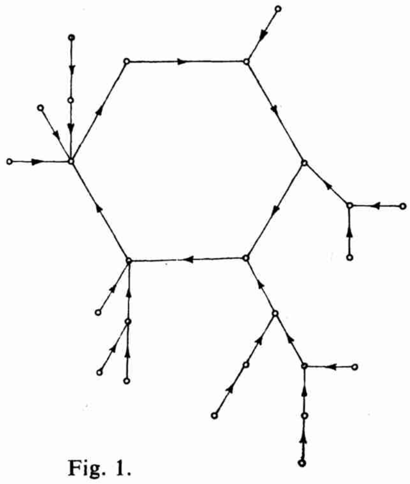

Fig. 1.