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$y > y_0$ can be proved similarly as in [4], p. 126, (14), if we consider the continuity properties of the function B . We have proved the relation (6), and Theorem 2 as well.

If we want to deduce Theorem 1 from Theorem 2, i.e., to verify that $g(x) = \sum_{n \leq x} \Lambda(n) \log(x/n) = x + O(x \log^{-k} x)$, $x \rightarrow \infty$, for any $k \in N$, we must put $A(x) = g(e^x)$ in Theorem 2 and show that in the halfplane $\operatorname{Re} s > 1$ the relation

$$-\frac{\zeta'(s)}{s^2 \zeta(s)} = \int_1^\infty \frac{g(x)}{x^{s+1}} dx = \int_0^\infty g(e^x) e^{-xs} dx$$

holds. It can be verified without any difficulties that the function $f(s) = -(\zeta'(s)/s^2 \zeta(s))$ satisfies all the assumptions of Theorem 2 for any $n \in N$; the integrability of the n -th derivative of the function $f(s) - 1/(s-1)$ was verified in the third section. According to Theorem 2 we have $e^{-x} g(e^x) = 1 + O(x^{-n})$, $x \rightarrow \infty$, i.e. $g(x) = x + O(x \log^{-n} x)$, $x \rightarrow \infty$, q.e.d.

It seems quite probable that no better estimation of the remainder term in the prime number theorem can be proved by the method used in this paper than $O(x \log^{-k} x)$, $x \rightarrow \infty$.

References

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