Lemma. *Each uniquely domatic graph with a domatic number at least* 2 *is connected.*

Proof. Let G be a disconnected graph with $d(G) \geqq 2$. Then each connected component of G has at least two vertices; otherwise the domatic number of G would be 1. Let $d(G) = d$, let $\{D_1, \ldots, D_d\}$ be a domatic partition of G. Let C be a connected component of G, let $V(C)$ be its vertex set. As each vertex of C can be adjacent only to vertices of C, we have $D_i \cap V(C) \neq \emptyset$ for each $i = 1, \ldots, d$ and $\{D_1 \cap V(C), \ldots \ldots, D_d \cap V(C)\}$ is a domatic partion of C. Put $D'_1 = (D_1 - V(C)) \cup (D_2 \cap V(C))$, $D'_2 = (D_2 - V(C)) \cup (D_1 \cap V(C))$, $D'_i = D_i$ for $i = 3, \ldots, d$. It is easy to prove that $\{D'_1, \ldots, D'_d)$ is a domatic partition of G different from $\{D_1, \ldots, D_d\}$ and hence G is not uniquely domatic.

Theorem 4. *A graph with the domatic number* 2 *is uniquely domatic, if and only if it is a star or a complete graph* K_2.

Proof. Let G be a uniquely domatic graph with the domatic number 2. By Lemma the graph G must be connected. If G is neither a star nor K_2, then there exists a spanning tree T of G which is neither a star nor K_2. Therefore there exists an edge e of T which joins two non-terminal vertices of T. Let T' and T'' be the connected components of the forest obtained from T by deleting e. None of the graphs T', T'' is an isolated vertex, therefore $d(T') = d(T'') = 2$. Let $\{D'_1, D'_2\}$ (or $\{D''_1, D''_2\}$) be a domatic partition of T' (or T'', respectively). It is easy to see that $\{D'_1 \cup D''_1, D'_2 \cup D''_2\}$ and $\{D'_1 \cup D''_2, D'_2 \cup D''_1\}$ are domatic partitions of T and also of G. These partitions are evidently different, which is a contradiction with the assumption that G is uniquely domatic. Therefore G must be either a star or K_2. On the other hand, the unique domatic partition of a star into two classes is such that one class consists only of the center and the other consists of all other vertices, because if a terminal vertex of a star belonged to the same class as the center, it would not be adjacent to a vertex of the other class. An analogous situation occurs in the case of K_2.

References

[1] *Cockayne, E. J. - Hedetniemi, S. T.:* Towards a theory of domination in graphs. Networks 7 (1977), 247—261.

[2] *Cockayne, E. J.:* Domination of undirected graphs — a survey. In: Theory and Applications of Graphs, Proc. Michigan 1976, ed. by Y. Alavi and D. R. Lick. Springer-Verlag Berlin—Heidelberg—New York 1978, pp. 141—147.

Authors address: 460 01 Liberec 1, Komenského 2, (Katedra matematiky Vysoké školy strojní a textilní).