Finally, the condition (13) implies that for every $\varepsilon > 0$ there is a μ, $0 < \mu < \varepsilon$, such that

$$(16) \qquad 0 < t_1, t_3 < \mu \Rightarrow \alpha(t_1, \varepsilon, t_3, \varepsilon) < \varepsilon - \mu .$$

To prove (16) we put $s = \limsup_{t_1, t_3 \to 0} \alpha(t_1, \varepsilon, t_3, \varepsilon)$. By (13) there is a $\bar{\mu} > 0$ such that for $0 < t_1, t_2 < \bar{\mu}$ we have $\alpha(t_1, \varepsilon, t_3, \varepsilon) < \varepsilon - \frac{1}{2}(\varepsilon - s)$. Evidently, $\mu = \min(\bar{\mu}, \frac{1}{2}(\varepsilon - s)$ satisfies the condition (16).

Now, setting $t_1 = d(x, y)$, $t_2 = d(x, Tx)$, $t_3 = d(y, Ty)$, $t_4 = \frac{1}{2}(d(x, Ty) + d(Tx, y))$ and taking into account (9) and (14)–(16) we see that all the assumptions of Theorem 1 are fulfilled. This completes the proof.

Remark 2. If α does not depend on t_2, t_3, t_4 then the condition (9) takes the form $d(Tx, Ty) \leqq \gamma(d(x, y))$, $x, y \in X$. Suppose that $\gamma(t) < t$ for $t > 0$ and γ is upper semicontinuous from the right. Then the conditions (10)–(13) are fulfilled, and, consequently, Theorem 2 implies the result of Boyd and Wong [1].

Theorem 2 generalizes also the results of S. Reich ([3], Th. 1) and C. S. Wong ([4], Th. 1).

Remark 3. In this paper "increasing" means nondecreasing. Note that in Theorem 2 the function α need not be increasing with respect to the first variable (cf. Remark 2).

The author thanks the referee for his valuable remarks.

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