$$\leqq \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_0^\infty e^{-((p+1)/t)\tau} \tau^p \|f(\tau) - f(t)\| \, d\tau =$$

$$= \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{Z_{t,p}} e^{-((p+1)/t)\tau} \tau^p \|f(\tau) - f(t)\| \, d\tau +$$

$$+ \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{R \setminus Z_{t,p}} e^{-((p+1)/t)\tau} \tau^p \|f(\tau) - f(t)\| \, d\tau \leqq$$

$$\leqq \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{Z_{t,p}} e^{-((p+1)/t)\tau} \tau^p \, d\tau \sup_{\tau \in Z_{t,p}} (\|f(\tau) - f(t)\| +$$

$$+ \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{R \setminus Z_{t,p}} e^{-((p+1)/t)\tau} \tau^p \, d\tau \|f(t)\| +$$

$$+ \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{R \setminus Z_{t,p}} e^{-((p+1)/t)\tau} \tau^p \|f(\tau)\| \, d\tau \leqq$$

$$= \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_0^\infty e^{-((p+1)/t)\tau} \tau^p \, d\tau \sup_{\tau \in Z_{t,p}} (\|f(\tau) - f(t)\|) +$$

$$+ \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{R \setminus Z_{t,p}} e^{-((p+1)/t)\tau} \tau^p \, d\tau \, \|f(t)\| +$$

$$+ M \frac{1}{p!}\left(\frac{p+1}{t}\right)^{p+1} \int_{R \setminus Z_{t,p}} e^{-((p+1)/t - \omega)\tau} \tau^p \, d\tau =$$

$$= \sup_{\tau \in Z_{t,p}} (\|f(\tau) - f(t)\|) + \frac{1}{\sqrt{(p+1)}} \|f(t)\| + M \frac{1}{\sqrt{(p+1)}} e^{7\omega t}$$

for every $t \in R^+$ and $p \in \{0, 1, \ldots\}$ such that $p + 1 > 2\omega t$.

It is clear that (1), (2) and (3) give the desired result.

A.4. Remark. The proof of Proposition A. 3 was inspired by a fascinating idea of W. Feller who used a probabilistic approach based on Chebyshev's inequality — see Chap. VII of [2].

Reference

[1] *Sova, M.:* Linear differential equations in Banach spaces, Rozpravy Československé akademie věd, Řada mat. a přír. věd, *85* (1975), No 6.

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