bridges of $H_n(k)$ for some k are the edges of the body of C and to each of them a unique k exists with this property. Thus C cannot be embedded into $H_n(k)$ for $k \notin K$. For $K = \emptyset$ the required tree is a star.

In [4], embedding rooted trees into rooted block graphs was defined. A graph is called rooted, if one of its vertices is fixed and called the root of the graph. By $H_n^*(k)$ for $k = 2, \ldots, n - 2$ we denote the graph consisting of two vertex-disjoint cliques, one with k, the other with $n - k$ vertices, with a bridge between them, which is rooted at a vertex of the clique with k vertices non-incident with the bridge.

Theorem 5. *Let T be a rooted tree with $n \geqq 4$ vertices. The tree T can be embedded into $H_n^*(k)$ for each $k = 2, \ldots, n - 2$ so that its root coincides with the root of $H_n^*(k)$ if and only if T is a snake whose root is a terminal vertex.*

Proof. Let T be a rooted snake with vertices $u_1, \ldots, u_n$ and edges $u_i u_{i+1}$ for $i = 1, \ldots, n - 1$. Let u_1 be its root. Then evidently T can be embedded into $H_n^*(k)$ for $k = 2, \ldots, n - 2$ in the required way so that the edge $u_k u_{k+1}$ is mapped onto the bridge of $H_n^*(k)$. Now suppose that T is a rooted tree whose root is not a terminal vertex. Then this root r has the degree at least 2. The tree T cannot be embedded into $H_n^*(2)$, because the root of $H_n^*(2)$ has the degree 1. Now let T be a rooted tree whose root r is its terminal vertex, but not a snake. Then T contains at least one vertex of a degree greater than 2; let u be such a vertex whose distance from r is minimal. Let $d(r, u) = h$. Then T cannot be embedded into $H_n^*(h + 1)$, because at this embedding all vertices of the path connecting r and u would have to be contained in the clique with $h + 1$ vertices and all vertices adjacent to u and not belonging to this path (they are at least two) would be embedded into the other clique and there would be at least two edges joining vertices of different cliques of $H_n(h + 1)$, which is impossible.

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