Proof. Let $R, R' \in \mathcal{Q}_d(\mathfrak{G})$, $R \subset R'$. Then the corresponding positive cones P, P' satisfy

$$P \cap -P = \{0\}, \quad P - P = G,$$

$$P' \cap -P' = \{0\}, \quad P' - P' = G,$$

$$P \subset P', \quad -P \subset -P',$$

and thus

$$P \cap -P' \subseteq P' \cap -P' = \{0\},$$

$$P + (-P') \supseteq P + (-P) = G.$$

Therefore $-P$ and $-P'$ are $\mathcal{P}(\mathfrak{G})$-complements of P and $-P' \supset -P$. This means that $\mathcal{P}(\mathfrak{G})$ is not modular, and so $\mathcal{Q}(\mathfrak{G})$ is not, either.

A group $\mathfrak{G}$ will be called an 0_d^*-group if each its directed order admits an extension to a linear one. For example, each 0^*-group (see [1]) is an 0_d^*-group.

Corollary 2.8.1. *Let $\mathfrak{G}$ be an 0_d^*-group and let the lattice $\mathcal{Q}(\mathfrak{G})$ be modular. Then each directed order of $\mathfrak{G}$ is linear.*

Proof. If there exist $R, R' \in \mathcal{Q}_d(\mathfrak{G})$, $R \subset R'$, then by proof of Theorem 2.8, $\mathcal{Q}(\mathfrak{G})$ is not modular. Therefore each $R \in \mathcal{Q}_d(\mathfrak{G})$ is a maximal order of G. And since each $R \in \mathcal{Q}_d(\mathfrak{G})$ admits an extension to a linear one, R is linear.

References

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