Consequently, (14) implies

$$(15)\quad \|f^{(q)}(t)\| \leqq \frac{q!}{2\pi}\, t\, \frac{\mu}{\mu+2} \int_0^{2\pi} \frac{N e^{t(\mu/\mu+2)}}{\left(t \dfrac{\mu}{\mu+2}\right)^{q+1}}\, d\tau = N e^{(\mu/\mu+2)t}\, \frac{q!\left(\dfrac{\mu+2}{\mu}\right)^q}{t^q}\,.$$

Taking $M = N$, $\omega = \mu/(\mu + 2)$, $\varrho = (\mu + 2)/\mu$ we get from (15) that (A) holds. The proof is complete.

11. Corollary. *Let F be a function defined on a subset of C with values in E. Then the following two statements are equivalent*:

(A) *there exist constants $N_1 \geqq 0$, $\varkappa_1 \geqq 0$ and $\mu_1 > 0$ so that*

(I) $\{z : \text{Re } z + \mu_1 |\text{Im } z| > \varkappa_1\}$ *lies in the domain of F*,

(II) *the function F is analytic in the domain* $\{z : \text{Re } z + \mu_1 |\text{Im } z| > \varkappa_1\}$,

(III) $\|F(z)\| \leqq N_1/(1 + |z|)$ *for every* $z \in C$, $\text{Re } z + \mu_1 |\text{Im } z| > \varkappa_1$;

(B) *there exist constants $N_2 \geqq 0$, $\varkappa_2 \geqq 0$ and $\mu_2 > 0$ and a function $\varphi \in \{z : : |\text{Im } z| < \mu_2 \text{ Re } z\} \to E$ so that*

(I) *φ is analytic in the domain* $\{z : |\text{Im } z| < \mu_2 \text{ Re } z\}$,

(II) $\|\varphi(z)\| \leqq N_2 e^{\varkappa_2 |z|}$ *for every* $z \in C$, $|\text{Im } z| < \mu_2 \text{ Re } z$,

(III) $\int_0^\infty e^{-\lambda\tau} \varphi(\tau)\, d\tau = F(\lambda)$ for sufficiently large $\lambda \in R$.

Proof. Immediate consequence of Theorems 8 and 10.

12. Remark. It is useful to compare Theorem 11 with theorems on generation of the so called holomorphic or analytic or parabolic semigroups as presented, e.g., in [1], [2], [3], [4] and [5].

For higher order linear differential equations in Banach spaces see also [6].

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Author's address: 115 67 Praha 1, Žitná 25 (Matematický ústav ČSAV).