One can easily show that this is equivalent to the conditions $a = b = c$ and $a^3 = 1$. Thus we conclude that the element $a \in F$ must be a root of the polynomial $x^2 + x + 1$ different from 1.

Theorem 4. *In an arbitrary desarguesian projective plane every six-fold homology of two triangles with no common vertex is equivalent to the special six-fold homology.*

Proof. According to the proof of the preceding theorem each pair of six-fold homologic triangles can be transformed by a certain automorphism of the plane onto triangles T_1, T_2 with the homogeneous coordinates described above. It is very easy to verify that this pair of triangles has the required property (cf. [4], [5]).

Remark. By an analogous argument we can obtain that in the desarguesian plane a six-fold perspectivity of two triangles with not common vertex implies their six-fold homology. Theorems 1—4 imply immediately:

Theorem 5. *A configuration* (H-T) *exists in a desarguesian projective plane over a field F if and only if in F there exists a root of the polynomial $x^2 + x + 1$ different from* 1.

Theorem 6. *If an arbitrary desarguesian-Fano plane contains an* (H-T) *configuration, then this plane has a finite subplane of order* 4. *In the case of finite Fano planes, they are exactly the projective planes over the Galois field of order* $n = 2^{2m}$. (cf. [1], [2].)

References

[1] *P. Dembowski:* Finite Geometries, Berlin—Heidelberg—New York, 1968.

[2] *M. Hall:* Combinatorial Theory, Walthamm—Toronto—London, 1967.

[3] *R. Hartshorne:* Foundations of Projective Geometry, New York, 1967.

[4] *K. Havlíček:* Zur Konstruktion einer endlichen Ebene, Čas. pěst. mat., *95* (1970), 71—75.

[5] *K. Havlíček, J. Tietze:* Zur Geometrie der endlichen Ebene der Ordnung $n = 4$, Czech. Mat. Journal *21* (*96*), 1971, 157—164.

Authors' address: Institute of Mathematics, Nicholas Copernicus University, Toruń, Poland.