sectional curvature given by the formula

$$K^0_{XJX} = - \|\text{grad Re} f(z)\|^2 . \tag{5.4}$$

Proof. Since f is holomorphic, Re f is harmonic in each variable. Therefore

$$(X^2 + (JX)^2) \text{ Re} f(z) = 0 \tag{5.5}$$

whenever X is a coordinate vector field. Moreover by linearity, (5.5) holds for any parallel vector field X on C^n. Let $s = \text{Re} f(z)$. Then we obtain (5.4) from (5.3) and (5.5).

Although Schur's lemma fails for the class H there does exist a curvature identity.

Theorem 5.5. *Let M be a Hermitian manifold with constant holomorphic sectional curvature μ. Then the curvature operator of M satisfies*

$$R_{WXWX} + R_{JWJXJWJX} - R_{WJXWJX} - R_{JWXJWX} = = 2\mu\{-\langle W, X\rangle^2 + \langle JW, X\rangle^2\} . \tag{5.6}$$

Proof. Let $X, Y \in \mathfrak{X}(M)$ be such that at a point $m \in M$ we have $\|X\| = \|Y\| = 1$, $\langle X, Y\rangle = 0$. Let $a^2 + b^2 = 1$. Substituting $aX + bY$ for X in (3.3) we find

$$R_{XJYXJY} + R_{JXYJXY} + 2R_{XJXYJY} - 2R_{XJYJXY} = 0 . \tag{5.7}$$

Similarly

$$R_{XYXY} + R_{JXJYJXJY} + 2R_{XJXYJY} + 2R_{XYJXJY} = 0 . \tag{5.8}$$

In [8] the following identity is proved for the class H

$$R_{ABCD} + R_{JAJBJCJD} - R_{JAJBCD} - R_{JABJCD} - R_{JABCJD} - - R_{AJBJCD} - R_{AJBCJD} - R_{ABJCJD} = 0 , \tag{5.9}$$

for $A, B, C, D \in \mathfrak{X}(M)$. From (5.7), (5.8), (5.9) it follows that

$$R_{XYXY} + R_{JXJYJXJY} = R_{XJYXJY} + R_{JXYJXY} . \tag{5.10}$$

Now we use the method of the proof of lemma 3.1 to derive (5.6) from (5.10).

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