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Summary

MORE ON SPANNING TREES OF CONNECTED GRAPHS

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If G is a finite undirected graph without loops and multiple edges denote by $k(G)$ the number of all its spanning trees. In [13] we defined $B_n^{(t)}$ to be the set of all positive integers y for which there exists a regular graph G of degree t on n vertices with $k(G) = y$. In the present paper the set $B_{12}^{(3)}$ is determined in detail (see p. 77). If $N(n)$ is the number of all connected cubic graphs on n vertices it is obvious that

$$|B_{2a}^{(3)}| \leqq N(2a)\,. \tag{1}$$

From [3] one can conclude that in (1) the equality holds for $a = 2, 3, 4$, and 5. On the other hand we have

$$|B_{12}^{(3)}| = 82\,, \quad N(12) = 85\,.$$

Fig. 1 shows two cubic graphs G_1, G_2 on 12 vertices with

$$k(G_1) = k(G_2) = 7280\,,$$

in Fig. 2 we have G_3, G_4 with

$$k(G_3) = k(G_4) = 8100\,,$$

and finally in Fig. 3 there are two graphs G_5, G_6 with

$$k(G_5) = k(G_6) = 8640\,.$$

Let $p(y, 2a)$ be the number of all cubic graphs G on $2a$ vertices with $k(G) = y$. Put

$$p(2a) = \max_{y \in B_{2a}{}^{(3)}} p(y, 2a)\,.$$

It can be shown that $p(6) = p(8) = p(10) = 1$, $p(12) = 2$.

Theorem 1. $\lim_{a \to \infty} p(2a) = \infty$.

In Fig. 6 we have two cubic graphs G_7, G_8 on 10 vertices each with

$$k(G_7) = 1599\,, \quad k(G_8) = 1600\,.$$

This example leads us to the following observation: For $a \geqq 3$ let us put

$$B_{2a}^{(3)} = \{y_1, y_2, y_3, \ldots, y_s\}$$

where

$$y_1 < y_2 < y_3 < \ldots < y_s$$

and let

$$f(a) = \min\,(y_i - y_{i-1})\,, \quad (i = 2, 3, \ldots, s)\,.$$

Theorem 2. *For* $a = 3, 4, 5, \ldots$ *we have*

$$f(a) \leqq 2a\,.$$

Theorem 3. *For the graph G illustrated by Fig. 7 we have*

$$k(G) = \frac{32}{\sqrt{3}}\left((2 + \sqrt{3})^{n+2} - (2 - \sqrt{3})^{n+2}\right).$$

In [16] and [17] we were concerned with the reconstruction of a connected graph from all its spannig trees. We proved that the complete bipartite graph $\langle 2, m\rangle$ and the wheels W_n are Uniquely Reconstructible from their Spanning Trees (URST-graphs). We also showed that the reconstruction is not always possible by giving finite and infinite counterexamples. R. D. Boyle [2] shows that the complete bipartite graph $\langle m, n\rangle$ is a URST-graph. In [5], [6], and [21] finite connected graphs G_0 with isomorphic spanning trees are characterized. It can be seen that G_0 are URST-graphs, too. Example 1 shows how to reconstruct G_0.

The last problem we are concerned with is that of reconstructing strong digraphs D from their spanning out-trees. An *out-tree* is a digraph with a source having no semicycles. Fig. 8 shows that D need not be uniquely reconstructible.

Theorem 4. *If n is a positive integer,* $n \geqq 4$, *then there exists a strong digraph not being uniquely reconstructible from its spanning out-trees.*