As in [7] we obtain the following upper bounds:

| $n =$ | Common upper bound for $cr_2(K_n)$ and $cr_1(K_n)$ |
|---|---|
| $6k = 3u$ | $u(u-2)(59u^2 - 98u + 24)/64 = n(n-6)(59n^2 - 294n + 216)/5184$ |
| $6k + 1 = 3u + 1$ | $u(177u^3 - 412u^2 + 180u + 64)/192 =$ <br> $= (n-1)(59n^3 - 589n^2 + 1541n - 435)/5184$ |
| $6k + 2 = 3u - 1$ | $(u-1)(177u^3 - 707u^2 + 727u - 117)/192 =$ <br> $= (n-2)(59n^3 - 530n^2 + 944n + 480)/5184$ |
| $6k + 3 = 3u$ | $(u-1)(59u^3 - 157u^2 + 45u - 27)/64 =$ <br> $= (n-3)(59n^3 - 471n^2 + 405n - 243)/5184$ |
| $6k + 4 = 3u + 1$ | $(u-1)(177u^3 - 235u^2 - 97u + 27)/192 =$ <br> $= (n-4)(59n^3 - 412n^2 + 356n + 240)/5184$ |
| $6k + 5 = 3u - 1$ | $(u-2)(177u^3 - 530u^2 + 416u - 96)/192 =$ <br> $= (n-5)(59n^3 - 353n^2 + 365n - 87)/5184$ |

To compare the upper bounds for the same graph (class of graphs) but for different surfaces it is useful to investigate the coefficients by the leading members:

| | $K_n$ | $K_{m,n}$ | $\bar{C}_n$ |
|---|---|---|---|
| Euclidean plane | $\frac{1}{64} = 0{\cdot}0156\ldots$ | $\frac{1}{16} = 0{\cdot}0625\ldots$ | $\frac{1}{64} = 0{\cdot}0156\ldots$ |
| Projective plane | $\frac{13}{1024} = 0{\cdot}0126\ldots$ | ? | ? |
| Torus | $\frac{59}{5184} = 0{\cdot}0113\ldots$ | $\frac{1}{24} = 0{\cdot}0416\ldots$ | ? |
| Klein bottle | $\frac{59}{5184} = 0{\cdot}0113\ldots$ | $\frac{1}{24} = 0{\cdot}0416\ldots$ | ? |

### *References*


[1] *Zarankiewicz, K.:* On a Problem of P. Turán concerning graphs. Fund. Math. *41* (1954), 137—145.

[2] *Urbanik, K.:* Solution du problème posé par P. Turán, Colloq. Math. *3* (1955), 200—201.

[3] *Guy, R. K.:* A combinatorial problem. Bull. Malayan Math. Soc. *7* (1960), 68—72.

[4] *Harary, F.* and *Hill, A.:* On the number of crossings in a complete graph. Proc. Edinburgh Math. Soc. (2), *13* (1962—3), 333—338.

[5] *Blažek, J.* and *Koman, M.:* A minimal problem concerning complete plane graphs. Theory of graphs and its applications, editor M. Fiedler, Proc. Symp. Smolenice June 1963, Prague 1964, 113—117.

[6] *Saaty, T. L.*: The minimum number of intersections in complete graphs. Proc. Nat. Acad. Sci., U.S.A., *52* (1964), 688—690.

[7] *Guy, R. K., Jenkyns, T. A., Schaer, J.*: The toroidal crossing number of complete graph. (Univ. Calgary, Research paper 18 (1967), 1—20.) J. Comb. Theory, *4*, (1968), 376—390.

[8] *Blažek, J.* and *Koman, M.*: On an extremal problem concerning graphs. CMUC, *8*, 1 (1967), 49—52.

[9] *Blažek, J.* and *Koman, M.*: The crossing number of complete $k$-chromatic graphs. (in Czech), Matemat., Sborník Ped., fak. UK Praha 1970, 69—84.

[10] *Harborth H.*: Über die Kreuzungszahl vollständiger, $n$-geteilter Graphen. Math. Nachrichten, *48* (1971), 179—188.

[11] *Guy, R. K.* and *Jenkyns, T. A.*: The toroidal crossing number of $K_{m,n}$. J. Comb. Theory, *6* (1969), 235—250.

[12] *Koman, M.*: On the crossing number of graphs, Acta Univ. Carol. Math. Phys., *10* (1969), 9—46.

[13] *Koman, M.*: A note on the crossing number of $K_{m,n}$ on the Klein bottle. Recent Advanc. in Graph Theory, Proc. Symp. Praque, June 1973, Editor M. Fiedler, Acad. Praque 1975, 327—334.

[14] *Guy, R. K.* and *Hill, A.*: The crossing number of the complement of a circuit. Discr. Math., *5* (1973), 335—344.

*Author's address:* 116 39 Praha 1, Rettigové 4 (Katedra matematiky a fyziky pedagogické fakulty UK).