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It follows from the previous definition that for each $x \in X$, $x \neq x_0$ we have

$$(1) \quad |g(x) - f(x_0)| \geq \frac{1}{3}\delta.$$

Evidently $g \in M(X)$. Further, $|g(x) - f(x)| < \frac{2}{3}\delta$ for all $x \in X$, hence

$$(2) \quad \|g - f\| \leq \frac{2}{3}\delta.$$

We shall show that

$$(a) \quad K(g, \frac{1}{9}\delta) \subset K(f, \delta);$$

$$(b) \quad K(g, \frac{1}{9}\delta) \cap S(X) = \emptyset.$$

It follows from (a), (b) by virtue of the well-known criterion of nowhere density (cf. [2], p. 37) that $S(X)$ is nowhere dense in $M(X)$.

Proof of (a). Let $h \in K(g, \frac{1}{9}\delta)$. Then using (2) we have

$$\|h - f\| \leq \|h - g\| + \|g - f\| < \frac{1}{9}\delta + \frac{2}{3}\delta < \delta.$$

Proof of (b). Let $h \in K(g, \frac{1}{9}\delta)$. Put $V = (h(x_0) - \frac{1}{9}\delta, h(x_0) + \frac{1}{9}\delta)$. Evidently $x_0 \in h^{-1}(V)$. We shall prove that $h^{-1}(V) = \{x_0\}$, hence $\text{Int } h^{-1}(V) = \emptyset$, therefore $h \notin S(X)$.

Suppose $x \in X$, $x \neq x_0$, $h(x) \in V$. Then $|g(x) - h(x)| < \frac{1}{9}\delta$, $|h(x) - h(x_0)| < \frac{1}{9}\delta$, $|h(x_0) - g(x_0)| < \frac{1}{9}\delta$, $g(x_0) = f(x_0)$ imply $|g(x) - f(x_0)| < \frac{1}{3}\delta$, which contradicts (1). This completes the proof.

References

- [1] K. R. Gentry - H. B. Hoyle, III: Somewhat continuous functions, Czechosl. Math. J. 21 (96), (1971), 5—12.
- [2] K. Kuratowski: Topologie I, PWN, Warszawa, 1958.
- [3] T. Šalát: Some generalizations of the notion of continuity and Denjoy property of functions, Čas. pěst. mat. 99 (1974), 380—385.

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