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# ON NOWHERE DENSITY OF THE CLASS OF SOMEWHAT CONTINUOUS FUNCTIONS IN $M(X)$

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This paper is closely related to the paper [3] and contains the solution of a problem formulated in [3].

Let $X$, $Y$ be two topological spaces. The function $f : X \to Y$ is said to be somewhat continuous on $X$ if for each set $G \subset Y$ open in $Y$ the following implication holds:

$$f^{-1}(G) \neq \emptyset \Rightarrow \operatorname{Int} f^{-1}(G) \neq \emptyset$$

(cf. [1]). This implies that every function $f : X \to Y$ continuous on $X$ is also somewhat continuous on $X$.

Let $X$ be a topological space, let $M(X)$ be the linear normed space (with the norm $\|f\| = \sup_{t \in X} |f(t)|$) of all real-valued functions which are defined and bounded on $X$. Denote by $S(X)$ and $C(X)$ the set of all $f \in M(X)$ which are somewhat continuous and continuous on $X$, respectively. A problem was posed in [3] wheter $S(X)$ is a nowhere dense subset of $M(X)$ provided that $S(X) \neq M(X)$.

We shall give an affirmative answer to the foregoing question.

Let us remark that if $X$ is a discrete space then each $f \in M(X)$ is continuous in $X$ and hence $M(X) = S(X) = C(X)$.

**Theorem.** *Let $X$ be a non discrete topological space. Then $S(X)$ is a nowhere dense subset of $M(X)$.*

Proof.*) If $f \in M(X)$, $\delta > 0$, put $K(f, \delta) = \{h \in M(X);\ \|h - f\| < \delta\}$. According to the assumption there exists an $x_0 \in X$ such that $\{x_0\}$ is not open in $X$. Given $f \in M(X)$, define a real-valued function $g$ on $X$ in the following way:

1) put $g(x_0) = f(x_0)$;

2) if $x \in X$, $x \neq x_0$, $|f(x) - f(x_0)| < \frac{1}{3}\delta$, put $g(x) = f(x_0) + \frac{1}{3}\delta$;

3) If $x \in X$, $|f(x) - f(x_0)| \geqq \frac{1}{3}\delta$, put $g(x) = f(x)$.

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