Proof. The method of variation of constants yields for the solution $y(t)$ of the initial problem (1), (2):

$$(20)\quad y(t) = x(t) + \int_{t_0}^{t} \frac{W_0(t, s)}{W(s)} f(s, y(s), \ldots, y^{(n-1)}(s), y[h(s)], \ldots, y^{(n-1)}[h(s)])\mathrm{d}s\,,$$

where $x(t)$ is the solution of (3) defined above and $W_0(t, s)$ is defined in (5).

Denote $\Gamma(t) = \max\limits_{t_0 \leqq s \leqq t} |x(s)|$ and $A(t) = \max\limits_{t_0 \leqq s \leqq t} \alpha_0(s)$. Then we obtain from (20) with respect to the assumptions of Theorem 3 the inequality

$$|y(t)| \leqq \Gamma(t) + n \int_{t_0}^{t} \frac{\alpha_0(t)\, D(s)}{W(s)} \psi(s)\, \omega(|y(s)|)\, \mathrm{d}s \leqq$$

$$\leqq \Gamma(t) + n\, A(t) \int_{t_0}^{t} \frac{D(s)}{W(s)} \psi(s)\, \omega(|y(s)|)\, \mathrm{d}s\,.$$

Let $[t_0, T)$ be an interval of existence of a solution $y(t)$ of the initial problem (1), (2). If we apply Lemma 2 to the last inequality for $t \in [t_0, T)$, we have (19).

According to (20), the derivatives $y^{(k)}(t)$, $k = 0, 1, \ldots, n - 1$ of the solution $y(t)$ of the initial problem (1), (2) satisfy

$$(21)\quad y^{(k)}(t) = x^{(k)}(t) + \int_{t_0}^{t} \frac{W_k(t, s)}{W(s)} f(s, y(s), \ldots, y^{(n-1)}(s), y[h(s)], \ldots$$

$$\ldots, y^{(n-1)}[h(s)])\, \mathrm{d}s\,.$$

Since (21) implies the inequality

$$|y^{(k)}(t)| \leqq |x^{(k)}(t)| + n \int_{t_0}^{t} \frac{\alpha_k(t)\, D(s)}{W(s)} \psi(s)\, \omega(|y(s)|)\, \mathrm{d}s\,,$$

$k = 0, 1, \ldots, n - 1$, the functions $y^{(k)}(t)$, $k = 0, 1, \ldots, n - 1$ are bounded on $[t_0, T)$ if $T < \infty$. With regard to Lemma 1 we conclude that the solution $y(t)$ of the initial problem (1), (2) exists for $t \in J$ and (19) holds. The proof is complete.

References

[1] *Bihari I.:* A generalization of a lemma of Bellman and its application to uniqueness problems of differential equation. Acta Math. Acad. Sci. Hung. 7 (1956), pp. 83—94.

[2] *Futák J.:* Über die Begrenzung und die Existenz der Lösungen der Differentialgleichung 4. Ordnung mit nacheilendem Argument. Acta Univ. Palac. Olomucensis, F.R.N., (in print).

[3] *Futák J.:* Existence and boundedness of solutions of the n-th order non-linear differential equation with delay. Práce a štúdie VŠD, č. 3, (in print).

Author's address: 010 88 Žilina, Marxa-Engelsa 25, (Vysoká škola dopravná).