struct the sequence $\mathscr{S}_0$ of the first co-ordinates of these vertices. This sequence is an infinite sequence of positive integers and no term is repeated in it, therefore it cannot be decreasing. Thus there are two terms p' and p'' of this sequence such that $p' < p''$ and p'' is the immediate successor of p' in $\mathscr{S}_0$. Obviously $p' > p^*$. This implies that the vertices $[p', q]$ and $[p'', q^*]$ are vertices of R_3 and there exists a finite dipath R_4 from $[p'', q^*]$ into $[p', q^*]$ such that each edge of R_4 is an edge of R_3; obviously we must take R_4 as a finite sequence whose ordering is inverse to the ordering of a subsequence of R_3; do not forget that the elements of sinking dipaths are written in the ordering in which they occur when going along such a dipath in the direction opposite to the orientation of edges, while at finite dipaths this is done inversely. Let $[p'', q^*] = u_0, u_1, \ldots, u_k = [p', q^*]$ be the sequence of vertices of R_4. There exist numbers $l_1, \ldots, l_{n-1}$ such that $u_0, \ldots, u_{l_1}$ are in P_{q^*}, the vertices $u_{l_i+1}, \ldots$ $\ldots, u_{l_{i+1}}$ are in P_{q^*+i} for $i = 1, \ldots, n-1$ and $u_{l_n+1}, \ldots, u_k$ are again in P_{q^*}. Let $u_{l_i} = [\tilde{p}_{i-1}, q^* + i - 1]$ for $i = 1, \ldots, n$, $u_{l_i+1} = [p_i, q^* + i]$ for $i = 1, \ldots, n$. Evidently $p_i = \tilde{p}_{i-1} - 1$ for $i = 1, \ldots, n$. Let U_1 be the set of all vertices $[p, q]$ such that either $p < p_i$, where $i \equiv q - q^* \pmod{n}$ and $q \neq q^*$, or $p < p''$, $q = q^*$. Let U_2 be the set of all vertices $[p, q]$ such that $p > \tilde{p}_i$, where $i \equiv q - q^* \pmod{n}$. Suppose that there exist vertices $x \in U_1$, $y \in U_2$ such that $\overrightarrow{xy}$ is an edge of G. Let $x = [p_x, q_x]$; then either $y = [p_x + 1, q_x]$ or $y = [p_x - 1, q_x + 1]$. First suppose $y = [p_x + 1, q_x]$. If $q_x = q^*$, then $p_x < p''$ because $x \in U_1$, but $p_x + 1 > \tilde{p}_0$ because $y \in U_2$; therefore $p_x < p'' \leqq \tilde{p}_0 < p_x + 1$. This is impossible because p_x, p'', p_0 are integers. If $q_x \neq q^*$, then $p_x < p_i$, where $i \equiv q_x - q^* \pmod{n}$ and $p_x + 1 > \tilde{p}_i$. But then $p_x < p_i \leqq \tilde{p}_i < p_x + 1$ and this is again impossible. Now let $y = [p_x - 1, q_x + 1]$. Then $p_x < p_i$, $p_x - 1 > \tilde{p}_j$, where $i \equiv q_x - q^* \pmod{n}$, $j \equiv q_x + 1 - q^* \pmod{n}$. This means $\tilde{p}_j < p_x - 1 < p_x < p_i$. But $\tilde{p}_j = p_i - 1$ and thus this inequality is also impossible. Now consider again the sourcing dipath R_1 whose vertex is $[p^*, q^*]$. The dipath R_1 being infinite, it must contain some vertices from U_2 because $V - U_2$ is a finite set. As $[p^*, q^*]$ is in U_1, there must exist an edge e of R_1 such that its initial vertex is in U_1 and its terminal vertex is in $V - U_1$. This terminal vertex cannot be in U_2, therefore it is in $V - (U_1 \cup U_2)$. But each vertex of $V - (U_1 \cup U_2)$ belongs to R_4 and therefore also to R_2. We see that R_1 has a common vertex with R. We have chosen a sourcing dipath R_1 and a sinking dipath R_2 quite arbitrarily and proved that they have a common vertex. Therefore each sourcing dipath and each sinking dipath in G have a common vertex. This implies the non-existence of a two-way infinite dipath in G; if it existed, then by deleting one edge from it we would obtain a sourcing dipath and a sinking dipath vertex-disjoint to each other, which would be a contradiction.

Reference

[1] *O. Ore:* Theory of Graphs. Providence 1962.

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