Having defined the sequence $\{\omega_n\}_{n=1}^{\infty}$, we put

$$\varphi(x) = x + \sum_{i=1}^{\infty} \omega_i \, \Psi_i(x) .$$

By (4), $\varphi'(x) = 1 + \sum_{i=1}^{\infty} \omega_i \, \Psi_i'(x) > 0$ for $x \in (0, 1)$. Since $\Psi_n(a_m) = 0$ for $n > 2m - 1$ and $\Psi_n(b_m) = 0$ for $n > 2m$, (5) implies that $\varphi(A) \subset D$ and $\varphi(B) \subset C$. The lemma is proved.

Now let f, g be as above two Pompeiu functions on R such that the sets $\hat{B} = \{x \in R : f'(x) > 0\}$, $D = \{x \in R : g'(x) > 0\}$ are dense in R. Denote $\hat{A} = \{x \in R : f'(x) = 0\}$, $C = \{x \in R, g'(x) = 0\}$. Let A and B be denumerable subsets of $\hat{A}$ and $\hat{B}$, respectively, which are dense in $(0, 1)$.

Define a function $k(x) = f(x) - g(\varphi(x))$ on $(0, 1)$, where φ is the function from Lemma 5. Then $k'(x) = f'(x) - g'(\varphi(x)) \, \varphi'(x) < 0$ for $x \in A$ and $k'(x) > 0$ for $x \in B$. Thus k is a Köpcke function.

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