

Werk

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Really, we have

$$\text{var} [\hat{\mathbf{Z}}_{p+N}(T) - \mathbf{Z}_{p+N+T}] = \text{var} \mathbf{e}_{p+N}(T)$$

and

$$\text{var} [\mathbf{Z}_{p+N}(T) - \mathbf{Z}_{p+N+T}] = \text{var} \mathbf{L} + \text{var} \mathbf{e}_{p+N}(T).$$

It leads to the desirable result, since $\text{var} \mathbf{L}$ is always a positive semidefinite matrix.

The influence of the random vectors $\mathbf{a}_p, \dots, \mathbf{a}_{p-q+1}$ on the $\hat{\mathbf{Z}}_{p+N}(T)$ can also be investigated in the same way as the scalar case.

Theorem 4. *If all the roots of the equation*

$$|\Theta(B)| = 0$$

are outside the unit circle, then $\mathbf{T}_j^N \rightarrow \mathbf{0}$ for $N \rightarrow \infty$.

Proof. Define

$$\mathbf{M} = \begin{pmatrix} \mathbf{0}, & \mathbf{I}, & \mathbf{0}, & \dots, & \mathbf{0} \\ \mathbf{0}, & \mathbf{0}, & \mathbf{I}, & \dots, & \mathbf{0} \\ \dots & \dots & \dots & \dots & \dots \\ \Theta_q, & \Theta_{q-1}, & \Theta_{q-2}, & \dots, & \Theta_1 \end{pmatrix}.$$

The roots of the matrix \mathbf{M} are identical with the roots of the polynomial

$$K(x) = |Ix^q - \Theta_1 x^{q-1} - \dots - \Theta_q| = |x^q \Theta(1/x)|$$

(see [1], p. 237). It implies that all the roots of the matrix \mathbf{M} are inside the unit circle and thus

$$\mathbf{M}^n \rightarrow \mathbf{0} \quad \text{for } n \rightarrow \infty.$$

Define

$$\Gamma_n = (\Theta_n^q, \Theta_{n+1}^{q-1}, \dots, \Theta_{n+q-1}^1)'$$

We have

$$\Gamma_{n+1} = \mathbf{M}^n \Gamma_1.$$

Because $\mathbf{M}^n \rightarrow \mathbf{0}$, also $\Theta_n^q \rightarrow \mathbf{0}$ for $n \rightarrow \infty$. Quite analogously as in the scalar case we can prove that $\mathbf{T}_j^N \rightarrow \mathbf{0}$.

References

- [1] *Anděl, J.*: Statistical analysis of time series, (Czech), SNTL, Praha 1976.
- [2] *Box G. E. P., Jenkins G. M.*: Time series analysis. Forecasting and control, Holden Day, San Francisco 1970.

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