In order to prove 2) choose $a \in V(G)$ such that $\mathrm{dp}(G_a) = n - 1$ (Theorem 2.8). Then $K_{n-1} \leqq G_a$ implies $K_n \leqq G$.

2.10. Theorem. *For every natural number n there exists a unique graph G_n with the following properties:*

1) *G_n has $2n$ vertices,*
2) $\mathrm{dp}(G_n) = n$.

The graphs G_n are given as follows: $G_0 = K_{\emptyset}$, $G_1 = (\{1, 2\}, \emptyset)$, $G_n = G_{n-1} + G_1$ *for all* $n \geqq 2$.

Proof. $\mathrm{dp}(G_n) = n$ by Theorem 2.7. Proof of the uniqueness of graphs G_n follows easily from the proof of Theorem 2.9.

2.11. Theorem. $\mathrm{dp}(G) \leqq 3$ *for every planar graph G.*

Proof. Let $\mathrm{dp}(G) = 4$. By Theorem 2.8 there exists a vertex $a \in V(G)$ such that $\mathrm{dp}(G_a) = 3$ and by the same argument there exists a vertex $b \in V(G_a)$ such that $\mathrm{dp}((G_a)_b) = 2$. But this means that $(G_a)_b$ contains a cycle C of length > 3 as its subgraph. But $C + K_2$ fails to be a planar graph.

2.12. Remark. There are examples of planar graphs depth 3. These are e.g. $C_k + G_2$ for any $k > 3/C_k$ is the cycle of length k and G_2 is that from Theorem 2.10).

Let us conclude the paper with a problem:

2.13. Neighborhood problem. Let G be a graph such that $\mathrm{dp}(G_a) = n \geqq 1$ for every vertex $a \in V(G)$. Does it follow $\mathrm{dp}(G) = n + 1$?

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