Consequently, $|p_m(z)| > |f(z) - p_m(z)|$ on C. But then since $f(z)$ has a zero in the disk D whose boundary is C, by Rouché's theorem [1, p. 157], it follows that $p_m(z)$ must also have at least one zero u in D. If $u \neq v$ then we take $k = m$ and $w = u$. If $u = v$ then let k be the smallest natural number larger than m such that $a_k \neq 0$. But then since $v \neq 0$, from the above it follows that $p_k(z)$ has a zero w in D such that $w \neq v$.

Next we prove:

Theorem. *Let* $\sum_{n=0}^{\infty} a_n z^n$ *be an entire transcendental function and* g *and* h *two distinct complex numbers. Let*

$$G = \{z \mid g = \sum_{n=0}^{k} a_n z^n \quad \textit{for some} \quad k < \infty\}$$

and

$$H = \{z \mid h = \sum_{n=0}^{k} a_n z^n \quad \textit{for some} \quad k < \infty\}$$

Then the set $G \cup H$ *has infinitely many accumulation points.*

Proof. Consider the entire transcendental function $f(z)$ given by (1). Since $h \neq g$, by Picard's big theorem [1, p. 341], at least one of the entire transcendental functions $f(z)$ or $-h + g + f(z)$ must have infinitely many distinct zeros. Without loss of generality, let $f(z)$ have infinitely many distinct zeros. But then, by the above Lemma, each such zero is an accumulation point of the set G mentioned in the Theorem.

Thus, the Theorem is proved.

The author thanks Prof. Stuart A. Nelson for helpful discussions.

Reference

1] *Saks, S.* and *Zygmund, A.*, Analytic Functions, Warsaw, 1952.

Author's address: Iowa State University Ames, Iowa 50011, U.S.A.