7.1. Proposition. *If* $\langle B, C\rangle$ *is decomposition of* $A \in \mathfrak{A}$, *then* $u[\mathscr{F}(B) \cdot \mathscr{F}(C)] = \mathscr{F}(A)$, *hence* $\mathscr{F}(B) \cdot \mathscr{F}(C)$ *is isomorphic to* $\mathscr{F}(A)$.

Proof. I. By definition (5.5), $L\mathscr{S}(B)$ and $L\mathscr{S}(C)$ generate the filters $\mathscr{F}(B)$ and $\mathscr{F}(C)$, respectively. Hence $L\mathscr{S}(B) \otimes L\mathscr{S}(C)$ generates $\mathscr{F}(B) \cdot \mathscr{F}(C)$. By 6.5, $u[L\mathscr{S}(B) \otimes L\mathscr{S}(C)] = L[\mathscr{S}(B) \odot \mathscr{S}(C)]$. By 6.3, $L[\mathscr{S}(B) \odot \mathscr{S}(C)] \subset L\mathscr{S}(A)$. Hence $L[\mathscr{S}(B) \odot \mathscr{S}(C)]$ generates $u[\mathscr{F}(B) \cdot \mathscr{F}(C)]$, and $u[\mathscr{F}(B) \cdot \mathscr{F}(C)] \subset \mathscr{F}(A)$. II. By 6.7, for every $P \in L\mathscr{S}(A)$ there exists a set $Q \in L[\mathscr{S}(B) \odot \mathscr{S}(C)]$ such that $Q \subset P$. Hence, for every $P \in \mathscr{F}(A)$ there exists a set $Q \in u[L\mathscr{S}(B) \otimes L\mathscr{S}(C)] \subset u[\mathscr{F}(B) \cdot \mathscr{F}(C)]$ such that $Q \subset P$. This implies $\mathscr{F}(A) \subset u[\mathscr{F}(B) \cdot \mathscr{F}(C)]$, which proves the proposition.

7.2. Theorem. *There exists a mapping* $\mathscr{F}$ *of the class* $\mathfrak{A}$ *of all dense linearly ordered sets with a first and with no last element into the class of all filters such that* (1) $\mathscr{F}(A)$ *is a filter on* eA, (2) *if* $\langle B, C\rangle$ *is a decomposition of* A, *then* $\mathscr{F}(B) \cdot \mathscr{F}(C)$ *is isomorphic to* $\mathscr{F}(A)$, (3) *if* A_1 *is isomorphic (as an ordered set) to* A_2, *then the filters* $\mathscr{F}(A_1)$, $\mathscr{F}(A_2)$ *are isomorphic. If* $A \in \mathfrak{A}$ *has a decomposition* $\langle B, C\rangle$ *such that* A, B, C *are mutually isomorphic, then* $\mathscr{F}(A)$ *is an idempotent filter.*

This follows at once from 7.1.

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