i.e.

$$-\bar{u}_k(t) + \bar{a}_k \cos k^2 t + \bar{b}_k \sin k^2 t = \bar{f}_k\,, \quad \bar{a}_k = \bar{p}_k\,, \quad \bar{b}_k = \bar{q}_k\,, \quad k \in S\,,$$

$$\int_0^\omega \{\bar{u}_k(t) \sum_{j\in S} j^2(z_j^0(t) + u_j^0(t))^2 + 2(z_k^0(t) + u_k^0(t)) \,.$$

$$. \sum_{j\in S} j^2(z_j^0(t) + u_j^0(t))\, \bar{u}_j(t)\} \cos k^2 t \,\mathrm{d}t = \frac{2}{\pi}\, \bar{p}_k\,, \quad k \in S$$

$$\int_0^\omega \{\bar{u}_k(t) \sum_{j\in S} j^2(z_j^0(t) + u_j^0(t))^2 + 2(z_k^0(t) + u_k^0(t)) +$$

$$+ \sum_{j\in S} j^2(z_j^0(t) + u_j^0(t))\, \bar{u}_j(t)\} \sin k^2 t \,\mathrm{d}t = \frac{2}{\pi}\, \bar{q}_k\,, \quad k \in S$$

has a unique solution for every $(\bar{f}, \bar{p}, \bar{q}) \in \mathcal{U} \times h^4 \times h^4$ satisfying

$$|\bar{u}|_4 + |\bar{a}|_{h^4} + |\bar{b}|_{h^4} \leqq C(|\bar{f}|_4 + |\bar{p}|_{h^4} + |\bar{q}|_{h^4})\,. \tag{23}$$

Obviously, it is sufficient to prove this assertion only for the last two equations and $\bar{a}_k$, $\bar{b}_k$, $k \in S$. Integrating we obtain equations (16) with

$$\bar{r}_k = 2\bar{p}_k\,, \quad \bar{s}_k = 2\bar{q}_k\,, \quad c_k = \bar{a}_k\,, \quad d_k = \bar{b}_k\,, \quad \sigma_k = k^2 g_k^2 \quad \text{for} \quad k = 1, 3\,, \quad \sigma_k = 0$$

for $k \neq 1, 3$,

From Lemma 3 it follows the existence and uniqueness of such $\bar{a}_k$, $\bar{b}_k$ and the estimate (23), which completes the proof.

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