group $\mathfrak{F}_\Delta$ and denoted by $\mathfrak{S}_\Delta/\mathfrak{F}_\Delta$. There exists a „natural" bijection between Δ and $\mathfrak{S}_\Delta/\mathfrak{F}_\Delta$.

Proof. The bijection can be constructed so that we assign each $\mathscr{L} \in \Delta$ all shifts of $\mathscr{L} = \mathscr{E}(\Delta) * \alpha$, that is, according to Theorem 5, exactly $\mathfrak{F}_\Delta \alpha$. ■

6. SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS

Let us apply the above considerations to the set Δ^* formed by all both-side oscillatory equations of the form $y'' = q(t)\, y$ on $(-\infty, \infty)$, $q \in C^0(\mathbf{R}, \mathbf{R})$, see [1], [2]. Δ^* is a subclass of the class of globally transformable second order homogeneous differential equations both-side oscillatory on arbitrary open (bounded or unbounded) intervals. If $\mathscr{E}(\Delta^*) \equiv y'' = -y$ on $(-\infty, \infty)$, then $\mathfrak{A}(\mathscr{E}(\Delta^*))$ is the fundamental group $\mathfrak{E}$, the stationary group $\mathfrak{A}(\mathscr{L})$ is the group of dispersions of the 1st kind of the equation $\mathscr{L} \in \Delta^*$ that is conjugate to the fundamental group $\mathfrak{E}$ (Theorem 4). $\mathfrak{A}^+_{\{E\}}(\mathscr{L})$ is the group of the central dispersions of the 1st kind of $\mathscr{L}$, both $\mathfrak{A}(\mathscr{L})$ and $\mathfrak{A}^+_{\{E\}}(\mathscr{L})$ being nontrivial, since each solution of $y'' = -y$ is periodic on $\mathbf{R}$, Theorems 1, 2, and 3. Each shift $\alpha \in \mathfrak{S}_{\Delta^*}$ with respect to $\mathscr{E}(\Delta^*)$ corresponds to a phase $f \in C^3(\mathbf{R}, \mathbf{R})$, $f'(t) \neq 0$ on $\mathbf{R}$, in the sense of formula $\alpha = \langle 1/\sqrt{|f'|}, f \rangle$, $1/\sqrt{|f'|}$ being the amplitude of α with the phase f.

We may introduce a group theoretical structure into the set $\mathfrak{S}_{\Delta^*}$ by the function composition rule for phases in distinction of the Brandt groupoid structure. This is the reason why we write $\alpha = \langle 1/\sqrt{|f'|}, f \rangle$ without the index of a specified equation. Then $\mathfrak{S}_{\Delta^*}/\mathfrak{F}_{\Delta^*} = \mathfrak{S}_{\Delta^*}/\mathfrak{E}$ is the right decomposition of the group of phases with respect to the fundamental (sub)group $\mathfrak{E}$, the elements of the decomposition being in 1-1 correspondence to the equations in Δ^* (Theorem 6). Theorem 5 describes all global Kummer transformations of an equation $(q_1) \in \Delta^*$ with a shift (phase) α into an equation $(q_2) \in \Delta^*$ with a shift (phase) β as elements of $\alpha^{-1}\mathfrak{E}\beta$.

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