Let for fixed k the sequence $\{n(k, i)\}$ of variable i be a sequence of natural numbers corresponding to $\varepsilon = 1^{-1}, \varepsilon = 2^{-1}, \ldots, \varepsilon = i^{-1}, \ldots$ and to uniform convergence of $\{f_n\}$ on A_k, i.e. for every i, for every $j > n(k, i)$ and for every $x \in A_k$ we have $|f_j(x)| < i^{-1}$. Obviously we can choose $\{n(k, i)\}$ to be increasing with respect to i. If k changes in the set of natural numbers, we obtain a double sequence $\{n(k, i)\}$. In virtue of the lemma there exists an increasing sequence $\{n(i)\}$ such that for every k and for every $i \geqq k$ $n(i) > n(k, i)$. Let $x = \Phi^{-1}(\{n(i)\})$. There exists a natural number k_0 such that $x \in A_{k_0}$. So for $i \geqq k_0$ we have $n(i) > n(k_0, i)$ and $|f_{n(i)}(x)| < < i^{-1}$ and simultaneously from the definition we have $f_{n(i)}(x) = i^{-1}$, a contradiction. The theorem is proved.

Corollary. *There exists a sequence of measurable real functions $\{f_n\}$ defined on $[0, 1]$, which tends to zero at every point and such that there does not exist a sequence $\{A_k\}$ of sets fulfilling $\bigcup_k A_k = [0, 1]$ such that the restricted sequence $\{f_n \mid A_k\}$ is uniformly convergent for every k.*

Proof. It suffices to take in the theorem the set $A \subset [0, 1]$ of the power of continuum and of measure zero and to define additionally $f_n(x) = 0$ for every n and for every $x \notin A$. Then we obtain a sequence of functions which are equal almost everywhere to zero and hence measurable.

References

[1] *Vrkoč, Ivo:* Remark about the relation between measurable and continuous functions, Čas. pro pěst. mat. *96* (1971) p. 225—228.

Author's address: 91-464 Lódź, ul. Zgierska 75/81, m. 226, Poland.