$g(x_0)\,u'$ and $g(x_0)\,u''$ are edges of H. Hence $E(H, g(x_0)) \cap E(H(s_1)) \neq \emptyset \neq$ $\neq E(H, g(x_0)) \cap E(H(s_2))$. Clearly, $E(H, g(x_0)) \subseteq E(H(s_1)) \cup E(H(s_2))$. From $(2_{H,h,J}/A_2)$ it follows that there exist $y^* \in E(H, g(x_0)) \cap E(H(s_1))$ and $y^{**} \in E(H, g(x_0)) \cap$ $\cap\, E(H(s_2))$ such that $h(y^*)$ and $h(y^{**})$ are adjacent in J. Denote $w_1 = h(y^*)$ and $w_2 = h(y^{**})$. It is obvious that $w_1 \in f^{-1}(s_1)$ and $w_2 \in f^{-1}(s_2)$.

We have proved that $(3_{J,f,G})$ holds. Hence G is a partition graph of J.

Now we assume that $(2_{G,g,H}/A_2)$ and $(2_{H,h,J}/A_3)$ hold. This implies that also $(2_{G,g,H}/A_1)$ and $(2_{H,h,J}/A_2)$ hold. Let r' be an arbitrary vertex of G. From $(2_{G,g,H}/A_2)$ it follows that $H(r')$ is nontrivial connected. From $(2_{H,h,J}/A_3)$ it follows that $J(r')$ is also connected. Since $V(J(r')) = f^{-1}(r')$, we have that $(4_{J,f,G})$ holds. Hence G is a contraction of J, which completes the proof.

Corollary. *Let G be a graph such that $\delta(G) \geqq 2$. Then G is a contraction of $L(L(G))$.*

Note that if G is a graph without a triangle which can be obtained from a cycle of a length at least six by adding one new edge, then G is not a partition graph of $L(G)$.

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