Let us define bijections $\pi : \mathscr{P}_1 \to \mathscr{P}_2$ and $\lambda : \mathscr{L}_1 \to \mathscr{L}_2$ such that $(x, y)^\pi = (x \circ_1 2, y)$, $[m, b]^\lambda = [m \circ_1 2, b]$, $(m)^\pi = (m \circ_1 2)$ and $[x]^\lambda = [x \circ_1 2]$. It is possible to verify that $x \circ_1 m = (x \circ_1 2) \circ_2 (m \circ_1 2)$ for all $x, m \in \mathscr{S}$. This statement together with the condition (a) of Theorem 1 imply the following results: if $(x, y) \mathscr{I}_1[m, b]$, then $(x, y)^\pi \mathscr{I}_2[m, b]^\lambda$. For the other couples of incident points and lines from $\mathscr{P}_{\mathbf{R}_1}$ the incidence in $\mathscr{P}_{\mathbf{R}_2}$ is obviously preserved. It means that (π, λ) is an isomorphism of the projective planes $\mathscr{P}_{\mathbf{R}_1}, \mathscr{P}_{\mathbf{R}_2}$. The following holds: $(\infty_1)^\pi = (\infty_2)$, $(O)^\pi = (O)$ and $[O]^\lambda = [O]$. The isomorphism (π, λ) satisfies the assumptions of Theorem 4.

If we construct an isotopism $(\alpha, \beta, \gamma, \delta)$ of PTR's $\mathbf{R}_1$ and $\mathbf{R}_2$ in the same way as in Theorem 4, then $x^\gamma = x \circ_1 2$, $x^\beta = x \circ_1 2$, $x^\alpha = x$ for all $x \in \mathscr{S}$. Since O is the zero in the PTR's $\mathbf{R}_1$ and $\mathbf{R}_2$, we have $\varphi_1 = \varphi_2 = \mathrm{id}_{\mathscr{S}}$. Then $\alpha = \delta = \mathrm{id}_{\mathscr{S}}$ and $\beta = \gamma$. Since (π, λ) is an isomorphism and $(x, y) \mathscr{I}_1[m, b] \Leftrightarrow (x, y)^\pi \mathscr{I}_2[m, b]^\lambda \Rightarrow (x^\beta, y) \mathscr{I}_2[m^\beta, b]$, it means that for each $x, y, m, b \in \mathscr{S}$ for which the above mentioned relation holds, $x \circ_1 m +_1 b = y$ and $x^\beta \circ_2 m^\beta +_2 b = y$. We can write $x \circ_1 m +_1 b = x^\beta \circ_2 m^\beta +_2 b$ which holds for each $x, m, b \in \mathscr{S}$, as can be verified from Tables 1—3. Hence the planar ternary rings $\mathbf{R}_1$ and $\mathbf{R}_2$ are isotopic.

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