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Kontakt/Contact

Digizeitschriften e.V.
SUB Göttingen
Platz der Göttinger Sieben 1
37073 Göttingen

✉ info@digizeitschriften.de

Then there exist two numbers $\bar{\varepsilon}_1 \in (0, \bar{\varepsilon}]$ and $\varrho > 0$ such that for every $\varepsilon \in (0, \bar{\varepsilon}_1]$ there is a unique $x^\varepsilon \in B(x_0, \varrho)$ satisfying ${}^eG(x^\varepsilon) = 0$. Moreover, $\lim_{\varepsilon \rightarrow 0} x^\varepsilon = x_0$.

Proof. Let $\bar{\varepsilon}_1$, α and ϱ be the numbers chosen in the proof of Lemma 3.3. Let $x_1^\varepsilon, x_2^\varepsilon \in B(x_0, \varrho)$, $0 < \varepsilon \leq \bar{\varepsilon}_1$ satisfy ${}^eG(x_1^\varepsilon) = {}^eG(x_2^\varepsilon) = 0$. Then we can write

$$x_1^\varepsilon - x_2^\varepsilon = -T^e({}^eG(x_1^\varepsilon) - {}^eG(x_2^\varepsilon) - {}^eG'(x_0)(x_1^\varepsilon - x_2^\varepsilon)).$$

Using [2] (relation 8.6.2), we obtain

$$\begin{aligned} \|x_1^\varepsilon - x_2^\varepsilon\|_X &\leq \bar{m}\|x_1^\varepsilon - x_2^\varepsilon\|_X \\ \cdot \sup \left\{ \|{}^eG'(x) - {}^eG'(x_0)\|_{[X,Y]} ; x \in B(x_0, \varrho), \varepsilon \in (0, \bar{\varepsilon}_1] \right\} &\leq \alpha\|x_1^\varepsilon - x_2^\varepsilon\|_X. \end{aligned}$$

As $\alpha < 1$, we have $x_1^\varepsilon = x_2^\varepsilon$. This completes the proof.

Lemma 3.5. Let X and Y be Banach spaces. Let $A \in [X, Y]$ and $B \in [Y, X]$ satisfy $AB = I_Y$. Then for every $\Delta \in [X, Y]$, $\|\Delta\|_{[X,Y]} \leq (2\|B\|_{[Y,X]})^{-1}$ there exists $B_\Delta \in [Y, X]$ such that

$$(3.10) \quad (A + \Delta)B_\Delta = I_Y,$$

$$(3.11) \quad \|B_\Delta\|_{[Y,X]} \leq 2\|B\|_{[Y,X]}.$$

If in addition the operators A and B fulfil $BA = I_X$, then B_Δ satisfies (3.10), (3.11) and $B_\Delta(A + \Delta) = I_X$.

Proof is easy.

References

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Author's address: 115 67 Praha 1, Žitná 25 (Matematický ústav ČSAV).