13. Remark. If for a harmonic function h and a set G the assumptions of Theorem 10 are satisfied and condition (ii) holds, the corresponding Radon measure μ is uniquely determined. Indeed, suppose that there are $\mu_1, \mu_2 \in C'$ such that $U\mu_1 = = U\mu_2 = h$ on G. Then for $\mu = \mu_1 - \mu_2$ $\mathscr{T}\mu = 0$, $U\mu = 0$ on G and by Theorem 26 of [13] $\mu = 0$, i.e. $\mu_1 = \mu_2$.

14. Corollary. *If h is a function harmonic on G and satisfying a Lipschitz condition, then there exists $\mu \in C'$ such that $h = U\mu$ on G.*

Proof. Note that $|\text{grad}\, h|$ is bounded on G, so that condition (ii) of Theorem 10 is satisfied.

15. Corollary. *If h is harmonic and bounded on G and* (ii) *from Theorem* 10 *is fulfilled, it follows that*

$$\int_G \text{grad}^2 h \, dH_m < \infty .$$

Proof. Choose $\varrho \in (0, 1)$. Then by the Gauss-Green theorem (compare [8], Remark 2.11)

$$\int_{G_\varrho} \text{grad}\, h \, . \, \text{grad}\, h \, dH_m = \left| \int_{\partial G_\varrho} h \frac{\partial h}{\partial n_\varrho} dH_{m-1} - \int_G h \, \Delta h \, dH_m \right| \leqq$$
$$\leqq \tilde{K} \sup |h(G)| < \infty \; ; \quad B := \tilde{K} \sup |h(G)| \, .$$

For $\varrho \nearrow 1$ we obtain

$$\int_G \text{grad}^2 h \, dH_m \leqq B \, .$$

In the following example, $G \neq \emptyset$ is an arbitrary bounded convex set. We are going to construct a harmonic function h, which does not satisfy the condition (ii) of Theorem 10.

16. Example. Let $x_0 \in \partial G$. By [6], Lemma 3.7, there is a continuous function h on $\bar{G} \setminus \{x_0\}$ which is strictly positive and harmonic on G and $h = 0$ on $\partial G \setminus \{x_0\}$. We shall prove that for such an h the condition (ii) does not hold.

Suppose on the contrary that (ii) holds and so Theorem 10 yields a Radon measure $\nu \in C'$ such that

$$U\nu = h \quad \text{on} \quad G \, .$$

Since $h = 0$ on $\partial G \setminus \{x_0\}$, we can show as in the proof of Theorem 26 in [13] that $\nu = 0$. But this is a contradiction with the fact that $h > 0$ on G.

References

[1] *H. Bauer:* Harmonische Räume und ihre Potentialtheorie, Springer Verlag, Berlin, 1966.

[2] *S. Dümmel:* On inverse problems for k-dimensional potentials, Nonlinear evolution equations and potential theory (pp. 73—93), Academia, Praha, 1975.

[3] *G. C. Evans:* The logarithmic potential (Discontinuous Dirichlet and Neumann problems), AMM Colloquium Publications, VI, New York, 1927.
[4] *G. A. Garrett:* Necessary and sufficient conditions for potentials of single and double layers, Amer. J. Math. *58* (1936), 95—129.
[5] *L. L. Helms:* Introduction to potential theory, Wiley-Interscience, New York, 1969.
[6] *R. A. Hunt* and *R. L. Wheeden:* Positive harmonic functions on Lipschitz domains, Trans. Amer. Math. Soc. *147* (1970), 507—527.
[7] *D. V. Kapànadze* and *I. N. Karcivadze:* Potentials in a domain with noncompact boundary (Russian), Thbilis. Sahelmc. Univ. Gamoqeneb. Math. Inst. Šrom. *2* (1969), 13—19.
[8] *J. Král:* The Fredholm method in potential theory, Trans. Amer. Math. Soc. *125* (1966), 511—547.
[9] *J. Král* and *J. Mařík:* Integration with respect to the Hausdorff measure over a smooth surface (Czech), Časopis Pěst. Mat. *89* (1964), 433—448.
[10] *J. Matyska:* Approximate differential and Federer normal, Czech. Math. J. *17* (*92*) (1967), 97—107.
[11] *I. Netuka:* Generalized Robin problem in potential theory, Czech. Math. J. *22* (*97*) (1972), 312—324.
[12] *I. Netuka:* An operator connected with the third boundary value problem in potential theory, Czech. Math. J. *22* (*97*) (1972), 462—489.
[13] *I. Netuka:* The third boundary value problem in potential theory, Czech. Math. J. *22* (*97*) (1972), 554—580.
[14] *I. Netuka:* Fredholm radius of a potential theoretic operator for convex sets, Časopis Pěst. Mat. *100* (1975), 374—383.
[15] *E. D. Solomencev:* Harmonic functions representable by Green's type integrals II (Russian), Izv. Akad. Nauk SSSR Ser. Mat. *31* (1967), 834—854.
[16] *Ch. de la Vallé Poussin:* Le potential logarithmique, Gauthier-Villars, Paris, 1949.

Author's address: 186 00 Praha 8, Sokolovská 83 (Matematicko-fyzikální fakulta UK).